Avenues: Math 1

Adapted from:
Mathematics Department Phillips
Exeter Academy Exeter, NH
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To the Student

We are grateful to the members of the Exeter Mathematics Department who have written the material in their original Math 1 Book. Avenues has adapted this curriculum to suit the needs of our students in some ways, but it stays true to the main goals of the Exeter Mathematics Department. As you work through it, you will discover that many topics in algebra have been integrated into a mathematical whole. There is no Chapter 5, nor is there a distinct section on one single topic. The curriculum is problem-based, rather than chapter-oriented.

A major goal of this course is to have you practice **thinking mathematically** and to learn to become a more independent and creative problem solver. Problem solving techniques, new concepts and algebra methods will become apparent as you work through the problems, and it is your classroom community's responsibility to make these conclusions together. Your responsibility is to keep appropriate notes for your records — there are no boxes containing important concepts. There is no index as such, but the <u>reference section at the end of the book</u> should help you recall the meanings of key words that are defined in the problems (where they usually appear italicized).

<u>The Mathematical Thinking Process</u>

1. Every night when you are attempting the problems on your assignment, you should be considering the **Mathematical Thinking Process** as outlined below:

    a. Stay/Think/Say/Draw
        i. Reading each question carefully and repeatedly is essential, especially since definitions, highlighted in italics, are routinely inserted into the problem texts. Check the reference section regularly.
        ii. It is important to make accurate diagrams whenever appropriate.

    b. Talk/Use Resources
        i. Talk out loud, speak to friends, ask questions, email your teacher, use Math Lab
        ii. Your prior knowledge – what you already know or have forgotten that you know – is your best resource.
        iii. Use your notes, the internet

    c. Estimate
        i. Before you try any mathematical formulas at all, you should have some idea of what the answer should be – is it really large like 3000? Or should it be something small like .05?

d. Mathematize
    i. Formulas, concepts, rules of mathematics that are prior knowledge you have can be used at this point in the process.
e. Try/Refine/Revise
    i. If something doesn't work, see why it didn't work
    ii. Change the method
    iii. Try something else!

1. Often it is necessary to rearrange an equation so that one variable is expressed in terms of others. For example, the equation $D = 3t$ expresses D in terms of t. To express t in terms of D, divide both sides of this equation by 3 to obtain $\frac{D}{3} = t$.

   a. Solve the equation $C = 2\pi r$ for r in terms of C.

   b. Solve the equation $p = 2w + 2h$ for w in terms of p and h.

   c. Solve the equation $3x - 2y = 6$ for y in terms of x.

2. Temperature is measured in both Celsius and Fahrenheit degrees. These two systems are of course related: the Fahrenheit temperature is obtained by adding 32 to 9/5 of the Celsius temperature. In the following questions, let C represent the Celsius temperature and F the Fahrenheit temperature.

   a. Write an equation that expresses F in terms of C.

   b. Use this equation to find the value of F that corresponds to $C = 20$.

   c. On the Celsius scale, water freezes at $0°$ and boils at $100°$. Use your formula to find the corresponding temperatures on the Fahrenheit scale. Do you recognize your answers?

3. (Continuation) A quick way to get an approximate Fahrenheit temperature from a Celsius temperature is to double the Celsius temperature and add 30. Explain why this is a good approximation. Convert 23°C the quick way. What is the difference between your answer and the correct value? For what Celsius temperature does the quick way give the correct value?

4. You measure your stride and find it to be 27 inches. If you were to walk to Columbia University, up Amsterdam Ave., which is about 4.5 miles, how many steps would you have to take? Remember that there are 12 inches in a foot, 3 feet in a yard, and 5280 feet in a mile.

5. The Millers must make a 120-km Thanksgiving trip to visit their grandparents. Pat Miller believes in driving at a steady rate of 80 kilometers per hour.
   a. How much time will it take Pat to make the trip?
   b. How many kilometers will the Millers travel in 18 minutes?
   c. Write an expression for the number of kilometers they will cover in t minutes of driving.
   d. After t minutes of driving, how many kilometers remain to be covered?

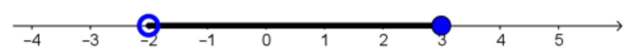
6. *Number-line graphs*. Observe the following useful notations, which may already be familiar:

   To indicate a *finite interval* on the number line, thicken that part of the number line and the two points that represent the numbers where the interval begins and ends. It is possible to represent an *infinite interval* (for example, ALL numbers greater than 3) by placing an endpoint on the beginning and thickening the number line in the respective direction and placing an arrow at the end on which you shaded.

   To indicate that an endpoint of an interval is included, place a solid dot on the number.

   To indicate that an endpoint is not included, place an open circle on the number.

   For example, the diagram at right illustrates those numbers that are greater than -2 and less than or equal to 3.



   Draw a number line for each of the following and indicate the numbers described:
   a. All numbers that are exactly two units from 5.
   b. All numbers that are more than two units from 5.
   c. All numbers that are greater than -1 and less than or equal to 7.
   d. All numbers that are less than four units from zero.

7. It is sometimes necessary to write fractions with variables in the denominator. Rewrite by hand each of the following as a single fraction. This is called *combining over a common denominator.*

   (a) $\dfrac{3}{a}+\dfrac{7}{a}$ **(b)** $\dfrac{3}{a}+\dfrac{7}{2a}$ **(c)** $\dfrac{3}{a}+\dfrac{7}{b}$ **(d)** $3+\dfrac{7}{b}$

8. Ryan earns x dollars every seven days. Write an expression for how much Ryan earns in one day. Ryan's spouse Lee is paid twice as much as Ryan. Write an expression for how much Lee earns in one day. Write an expression for their combined daily earnings.

9. On a number line, graph all numbers that are closer to 5 than they are to 8.

10. Remy walked to a friend's house, m miles away, at an average rate of 4 mph. The m-mile walk home was at only 3 mph, however. First, express the time Remy spent walking to their friends house as a fraction. Then express the time Remy spent walking home as a fraction; Finally, express the total time Remy spent walking as a fraction.

11. The sum of four *consecutive integers* is 2174. What are the integers?

12. (*Continuation*) The smallest of four consecutive integers is n. What expression represents the next larger integer? Write an expression for the sum of four consecutive integers, the smallest of which is n. Write an equation that states that the sum of four consecutive integers is s. Solve the equation for n in terms of s. Check that your answer to the previous question satisfies this equation by considering the case $s = 2174$.

13. Graph on a number line the intervals corresponding to these two signs on the highway.
    a. The maximum speed is 65 mph and the minimum speed is 45 mph.
    b. The maximum speed is 55 mph.

14. Solve for x: **(a)** $2(x-1) = 3(x+2)$ **(b)** $-4(2x-2) = 3(x+1)$

15. On a number line, graph a number that is twice as far from 5 as it is from 8. How many such numbers are there?

16. Solve the equation $A = P + Prt$ for r. Solve the equation $A = P + Prt$ for P.

17. Draw a number line for each of the following and indicate the numbers described (if any):
    a. The numbers that are less than 2 or greater than 4.
    b. The numbers that are less than 2 and greater than 4.

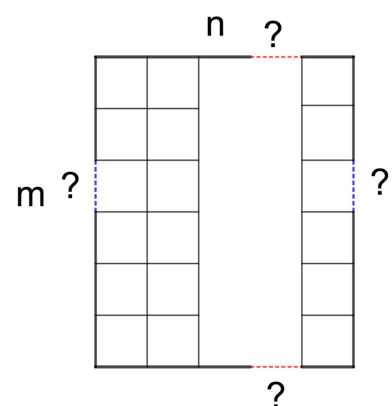
18. Find the smallest positive integer divisible by every positive integer less than or equal to 10.

19. Graph on a number line the intervals described below:
    a. All numbers that are greater than 1 or less than 3.
    b. All numbers that are greater than -5 and less than or equal to 4.
    c. All numbers whose squares are greater than or equal to 1.
    d. How would you write each of these (a-c) with an inequality?

20. Randy and Sandy have a total of 20 books between them. After Sandy loses three by leaving them on the bus, and some birthday gifts double Randy's collection, their total increases to 30 books. How many books did each have before these changes?

21. Do each of the following:
    a. On a number line, graph $x < 2$.
    b. On the same line, graph $x + 5 < 2$; how does the location of the new graph relate to the graph of $x < 2$?
    c. On the same line, graph $x - 3 < 2$; how does the location of the new graph relate to the graph of $x < 2$?

22. A flat, rectangular board is built by gluing together a number of square pieces of the same size. The board is *m* squares wide and *n* squares long.
    a. Using the letters *m* and *n*, write expressions for the total number of 1x1 squares;
    b. the total number of 1x1 squares with free edges (the number of 1x1 squares that are not completely surrounded by other squares);
    c. the number of completely surrounded 1x1 squares;
    d. the perimeter of the figure.



23. Using the variable x to represent a certain number, write an algebraic expression to represent each of the following:
    a. Eleven more than one third of the number.
    b. Three times the difference between the number and twelve.
    c. Two times the number, decreased by the sum of the number squared and two.

24. By hand, combine the following over a common denominator.

(a) $\frac{1}{4}+\frac{1}{5}$ **(b)** $\frac{1}{10}+\frac{1}{11}$ **(c)** $\frac{1}{x}+\frac{1}{x+1}$

25. (*Continuation*) Evaluate your answer to (c) with $x = 4$ and then with $x = 10$. How do these answers compare to your answers to (a) and (b)?

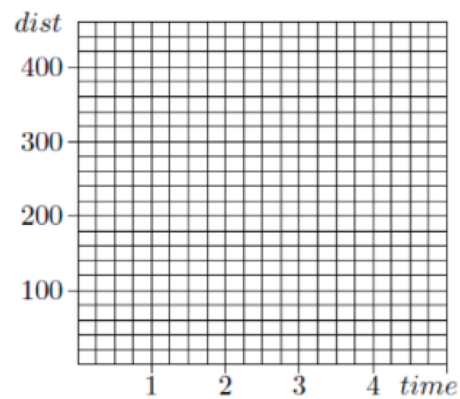
26. Which of the following seven expressions does not belong in the list?

$a-b+c \quad c-b+a \quad c-(b-a) \quad -b+a+c \quad a-(b-c) \quad b-(c-a) \quad a+c-b$

27. Forrest is texting while driving along the freeway at 70 miles per hour. How many feet does the car travel during the 3-second interval when Forrest's eyes are not on the road?

28. The statement "x is between 13 and 23" defines an interval using two simultaneous inequalities: $13 < x$ *and* $x < 23$. The statement "x is not between 13 and 23" also uses two inequalities, but they are *non*-simultaneous: $x < 13$ and $x > 23$. Graph these two examples on a number line. Notice that there is a compact form $13 < x < 23$ for only one of them.

29. Crossing a long stretch of the Canadian plains, passenger trains maintain a steady speed of 80 mph. At that speed, what distance is covered in half an hour? How much time is needed to cover 200 miles? Fill in the missing entries in the table below, and plot points on the grid at right.



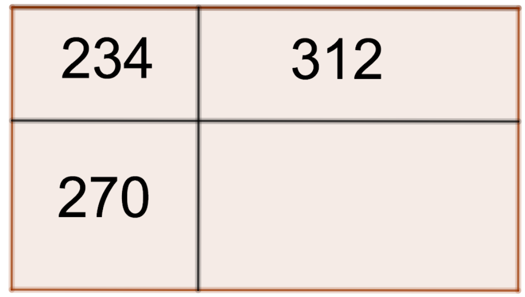
Time	0	1/2		1	2		3		4	t
distance			60			200		300		

30. (*Continuation*) The Canadian plains problem contains relationships that is called *direct variation.* In your own words, describe what it means for one quantity to *vary directly* with another. Which of the following describe direct variations?
    a. The height of a ball and the number of seconds since it was thrown.
    b. The length of a side of a square and the perimeter of the square.
    c. The length of a side of a square and the area of the square.
    d. The gallons of water in a tub and the number of minutes since the fawcett was opened.

31. (*Continuation*) Sketch graphs for each of the situations described above. Be sure to include meaningful descriptions and scales for each axis.

32. Remy walked to a friend's house, m miles away, at an average rate of 4 mph. The m-mile walk home was only at 3 mph. Remy spent 2 hours walking in all. Find the value of m.

33. The rectangle shown at right has been broken into four smaller rectangles. The areas of three of the smaller rectangles are shown in the diagram. Find the area of the fourth one.



34. Tickets to a school play cost \$2 if bought in advance and \$3 at the door. By selling all 400 tickets, \$1030 was collected. Let x represent the number of tickets sold at the door.
    a. In terms of x, how many tickets were sold in advance?
    b. In terms of x, how much money was taken in by the tickets sold in advance?
    c. Write and solve an equation to find out how many tickets were sold in advance.

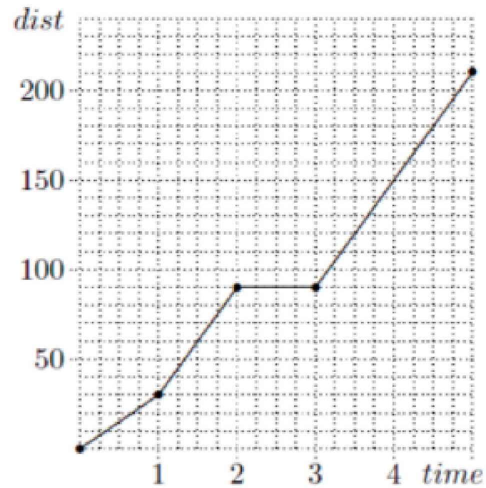
35. Chandler was given \$75 for a birthday present. This present, along with earnings from a summer job, is being set aside for a mountain bike. The job pays \$6 per hour, and the bike costs \$345. To be able to buy the bike, how many hours does Chandler need to work?

36. (*Continuation*) Let h be the number of hours that Chandler works. What quantity is represented by the expression $6h$? What quantity is represented by the expression $6h + 75$?
    a. Graph the solutions to the inequality $6h + 75 \geq 345$ on a number line.
    b. Graph the solutions to the inequality $6h + 75 < 345$ on a number line.
    c. What do the solutions to the inequality $6h + 75 \geq 345$ signify?

37. If you bike 10 miles from Avenues to Columbia University in 40 minutes, you will most likely not be traveling at a constant rate. But if you did, what rate would it be? This value is your *average speed* for the trip.

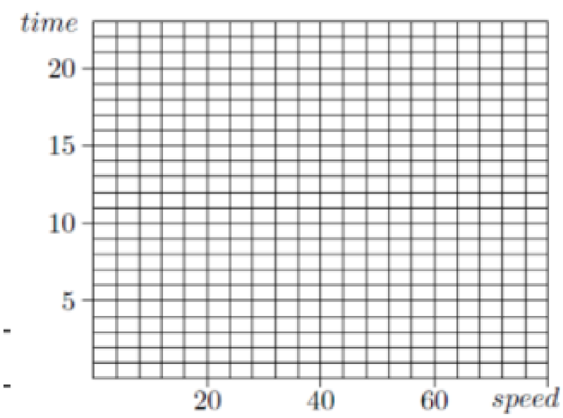
38. One day, you rode your bike to the beach. You pedal hard for the first ten minutes and cover 4 miles. Tired, you slow down and cover the last 6 miles in 36 minutes. What is your average speed for the return trip?

39. Sandy recently made a 210-mile car trip, starting from home at noon. The graph at right shows how Sandy's distance from home (measured in miles) depends on the number of hours after noon. Make up a story that accounts for the four distinct parts of the graph. In particular, identify the speed at which Sandy spent most of the afternoon driving.



40. Solve the inequality $3 - x > 5$ using only the operations of addition and subtraction. Is $x = 0$ a solution to the inequality?

41. To do a college visit, Trace must make a 240-mile trip by car. The time required to complete the trip depends on the speed at which Trace drives, of course, as the table below shows. Fill in the missing entries, and plot points on the grid provided. Do the quantities time and speed vary directly? It makes sense to connect your plotted points with a *continuous* graph. Explain why.

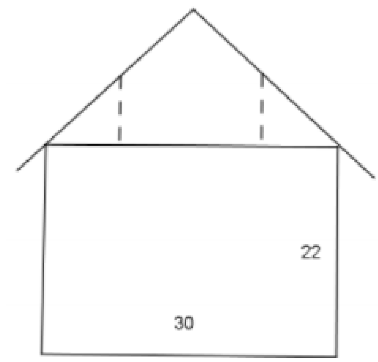


Speed	15	20	25			48		60		r
time				8	6		4.8		3	

42. Pat bought several pens at Walgreens, for 60 cents each. Spending the same amount of money at CVS, Pat then bought some pens that cost 80 cents each. In all, 42 pens were bought. How many pens did Pat buy CVS?

43. A small pool is 20 feet long, 12 feet wide and 4 feet deep. There are 7.5 gallons of water in every cubic foot. At the rate of 5 gallons per minute, how long will it take to fill this pool?

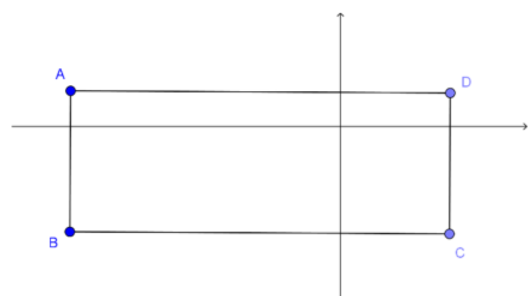
44. A town's building code does not permit building a house that is more than 35 feet tall. An architect working on the design shown at right would like the roof to be sloped so that it rises 10 inches for each foot of horizontal length. Given the other dimensions in the diagram, will the builder be allowed to carry out this plan?



45. (*Continuation*) Two vertical supports (shown dotted in the diagram) are to be placed 6 feet from the center of the building. How long should they be?

46. Jan walks 2 miles at a constant rate of 3 miles per hour and then runs 1 mile at a constant rate of 8 miles per hour.
    a. What is Jan's average speed for the entire trip?
    b. Is the average speed in part (a) equal to the average of Jan's two speeds?

47. The rectangle *ABCD* shown at right has sides that are parallel to the coordinate axes. Side *AD* is three times the length of side *AB* and the perimeter of *ABCD* is 56 units. (Figure not to scale).
    a. Find the dimensions of *ABCD*.
    b. Given the information $D = (9, 2)$, find the coordinates for points *A*, *B*, and *C*.



48. A ladder is leaning against the side of a building. Each time I step from one rung to the next, my foot moves 6 inches closer to the building and 8 inches further from the ground. The base of the ladder is 9 ft. from the wall. How far up the wall does the ladder reach?

49. Solve the following for x:

(a) $\dfrac{x}{2}+\dfrac{x}{5}=6$ **(b)** $\dfrac{x}{3}+\dfrac{x+1}{6}=4$ **(c)** $\dfrac{x}{5}-\dfrac{x+2}{10}=1$

50. In 1986, renovations were made to the stairs in the Statue of Liberty in an overall celebration of its bicentennial. Francesco Schettino was the draftsman in charge of the calculations for the details of the structural steel for those stairs. He found that the stairs in the Lady's pedestal needed to have a vertical rise of 7.8 inches and a horizontal run of 9.75 inches. The more complicated spiral stairs that led up the body had a constant vertical rise of 9 inches and average horizontal run of 5.06 inches.
    a. Using graph paper, create a graphical representation of these two sets of stairs.
    b. Which flight of stairs do you think is steeper? Why?
    c. Calculate the ratio for each flight and verify that the greater ratio belongs to the flight you thought was steeper.

51. (*Continuation*) On a coordinate plane, the *slope* of a line is a numerical measure of how steep the line is. It is calculated by dividing the change in *y*-coordinates by the corresponding change in *x*-coordinates between two points on the line:

$$slope = \frac{\text{difference between y-coordinates}}{\text{difference between x-coordinates}}$$

Plot the points (1, 3) and (7, 6). and then calculate the slope of the line that goes through them. Calculate the slope of the line that goes through the two points (0, 0) and (9, 6). Which line is steeper?

52. Explain why the descriptions "right 5 up 2", "right 10 up 4", "left 5 down 2", "right 5/2 up 1", and "left 1 down 2/5" all describe the same inclination for a straight line.

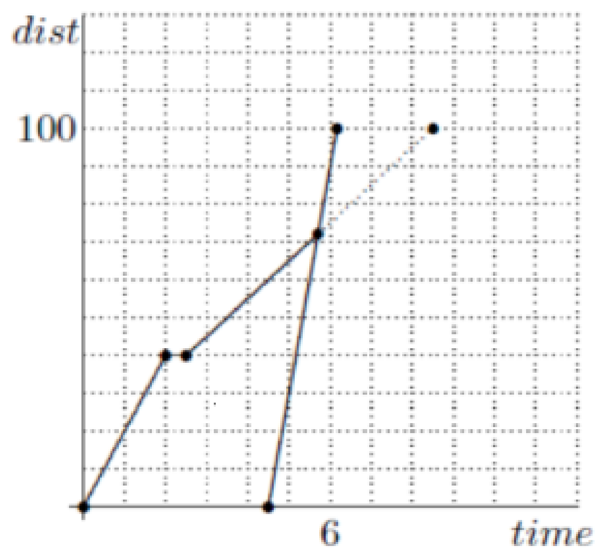
53. In August, 2011, the Saw Mill River in Yonkers, was hit by hurricane Irene. At noon one day that August the water in the Saw Mill River was 9.6 feet above flood stage (the water level where the river would normally begin to flood!). It then began to recede, its depth dropping at 4 inches per hour.
    a. At 3:30 that afternoon, how many inches above flood stage was the river?
    b. Let *t* stand for the number of hours since noon, and *h* stand for the corresponding number of inches that the river was above flood stage. Make a table of values, and write an equation that expresses *h* in terms of *t*.
    c. Plot *h versus t*, putting *t* on the horizontal axis.
    d. For how many hours past noon was the river at least 36 inches above flood stage?

54. A sign placed at the top of a hill on route 684 north of White Plains says "8% grade. Trucks use lower gear." What do you think that "8% grade" might mean?

55. Cass decided to sell game programs for the next Avenues soccer game. The printing cost was 20 cents per program, with a selling price of 50 cents each. Cass sold all but 50 of the programs printed, and made a profit of $65. Let p represent the number of programs printed.
    a. Write an expression for the amount of programs sold.
    b. Write an expression to calculate the profit on the programs sold.
    c. Set up an equation that relates p with the cost to print and sell the programs and the total profit made. See if you can solve it.

56. Draw the segment from (3, 1) to (5, 6), and the segment from (0, 5) to (2, 0). Calculate their slopes. You should notice that the segments are equally steep, and yet they differ in a significant way. Do your slope calculations reflect this difference?

57. At noon one day, Alex decided to take a long bike ride from Manhattan to Jones Beach State Park on Long Island, a distance of 100 km. Later in the day, the rest of the family packed into their van and drove to the park along Alex's bike route. They overtook Alex after driving for 1.2 hrs, stopped long enough to put Alex and bicycle in the van, and continued to the Jones Beach. Refer to the graph as you answer the following questions about the day's events:



a. Alex pedaled at two different rates during the biking part of the trip. What were they?
b. After biking for a while, Alex stopped to rest. How far from home was Alex then? For how long did Alex rest?
c. How far from home was Alex when the family caught up?
d. At what time did the family arrive at Jones Beach?
e. At what time would Alex have arrived, if left to bicycle all the way?
f. What distance separated Alex and the rest of the family at 5 pm?

58. Robin and Wes are solving the inequality $132 - 4x \le 36$. Each begins by subtracting 132 from both sides to get $-4x \le -96$, and then each divides both sides by -4. Robin gets $x \le 24$ and Wes gets $x \ge 24$, however. Always happy to offer advice, Alex now suggests to Robin and Wes that answers to inequalities can often be checked by substituting $x = 0$ into both the original inequality and the answer. What do you think of this advice? Graph each of these answers on a number line.

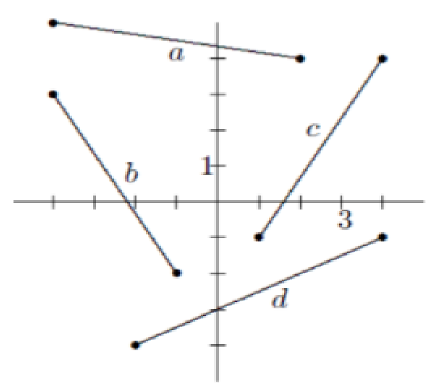
59. (*Continuation*) After hearing Alex's suggestion about using a test value to check an inequality, Cameron suggests that the problem could have been done by solving the equation $132 - 4x = 36$ first. Explain the reasoning behind this strategy.

60. (*Continuation*) Tracy, who has been keeping quiet during the discussion, remarks, "The only really tricky thing about inequalities is when you try to multiply them or divide them by negative numbers, but this kind of step can be avoided altogether. Cameron just told us one way to avoid it, and there is another way, too." Explain this remark by Tracy.

61. Solve the following inequality for x: $2(1 - 3x) - (x - 5) > 1$

62. Each beat of your heart pumps approximately 0.06 liter of blood.
    a. If your heart beats 50 times, how much blood is pumped?
    b. How many beats does it take for your heart to pump 0.48 liters?

63. (*Continuation*) Direct-variation equations can be written in the form $y = kx$, and it is customary to say that *y depends on x* or *y varies directly as x*. Find an equation that shows how the volume V pumped depends on the number of beats n. Graph this equation, using an appropriate scale, and calculate its slope. What does the slope represent in this context?



64. Estimate the slopes of all the segments in the diagram. Identify those whose slopes are negative. Find words to characterize lines that have negative slopes.

65. Find the slope of the line containing the points $(4, 7)$ and $(6, 11)$. Find coordinates for another point that lies on the same line and be prepared to discuss the method you used to find them.

66. To earn Hall of Fame Distinction at Avenues, a girl on the cross-country team must run the 5-km course in less than 20 minutes. What is the average speed of a 20 minute runner, in km per hour? In meters per second? Express your answers rounded to two decimal places.

67. (Continuation) The *proportion* $\dfrac{5}{20} = \dfrac{x}{60}$ is helpful for the previous question. Explain this proportion, and assign units to all four of its members.

68. One day in October at 9am, Sam began hiking an 8-mile trail, hiking for 2.5 hours at a pace of 2 miles per hour, and then stopping for half an hour to enjoy the view and have a snack. Sam then hiked the remainder of the trail at 3 miles per hour. Meanwhile, Jaden decided to run the same trail. Jaden began 1.5 hours after Sam began hiking, and ran at a rate of 7 miles per hour.
    a. Draw a graph with time (in hours since Sam began) on the horizontal axis and distance (in miles) on the vertical axis.
    b. According to your graph, does Jaden catch up with Sam on the trail? If so, is it before, during, or after Sam's snack break?
    c. What time did Sam reach the end of the trail?

69. The stretch of a spring varies directly as the weight attached to the spring. If a weight of 20 grams stretches a spring 5 cm, what weight would stretch the spring 8 cm?

70. (*Continuation*) Recall that direct variation equations can be written in the form $y = kx$, and it is customary to say that *y depends on x*. Find an equation that shows how the stretch of the spring, d, depends on the weight, w. Graph your equation by hand. What does the slope of the line represent in this context?

71. Alex was hired to unpack and clean 576 very small items of glassware, at five cents per piece successfully unpacked. For every item broken during the process, however, Alex had to pay \$1.98. At the end of the job, Alex received \$22.71. How many items did Alex break?

72. Suppose that n represents a positive even integer. What expression represents the next even integer? the next odd integer? I am thinking of three consecutive even integers whose sum is 204. What are they?

73. A car and a small truck started out from Manhattan at 8:00 am. Their distances, in miles, from Manhattan, recorded at hourly intervals, are recorded in the tables at right. Plot this information on the same set of axes and draw two lines connecting the points in each set of data. What is the slope of each line? What is the meaning of these slopes in the context of this problem.

time	*car*	*truck*
8:00	0	0
9:00	52	46
10:00	104	92
11:00	156	138
12:00	208	184

74. (*Continuation*) Let t be the number of hours each vehicle has been traveling since 8:00 am (thus $t = 0$ means 8:00 am), and let d be the number of miles traveled after t hours. For each vehicle, write an equation relating d and t.

75. Chris does a lot of babysitting. When parents drop off their children and Chris can supervise at home, the hourly rate is \$3. If Chris has to travel to the child's home, there is a fixed charge of \$5 for transportation in addition to the \$3 hourly rate.

    a. Using desmos.com, graph $y = 3x$ and $y = 3x + 5$.
    b. What do these lines have to do with the babysitting context?
    c. What features do they have in common? How do they differ?
    d. What does the graph of $y = 3x + 6$ look like? What change in the babysitting context does this line suggest?

76. If k stands for an integer, then is it possible for $k^2 + k$ to stand for an odd integer? Be prepared to justify your answer.

77. Try to think of a number k for which $k^2 < k$ is true? Graph all such numbers on a number line. Also describe them using words, and using algebraic notation.

78. One year after Robin deposits 400 dollars in a savings account that pays r % annual interest, how much money is in the account? Write an expression using the variable r.

79. Solve $\frac{x}{4} + \frac{x+1}{3} \le \frac{1}{2}$ and shade the solution interval on a number line the y-axis. These points are called the *x-intercept* and *y-intercept*, respectively. Use these points to make a quick sketch of the line.

80. How much time does it take for a jet to go 119 miles, if its speed is 420 mph? Be sure to specify the units for your answer.

81. *Word chains*. As the ancient alchemists hoped, it is possible to turn *lead* into *gold*. You change one letter at a time, always spelling real words: lead—load—toad—told—gold. Using the same technique, show how to turn *work* into *play.*

82. Compare the graph of $y = 2x + 5$ with the graph of $y = 3x + 5$.
    a. Describe a context from which the equations might emerge
    b. Linear equations that look like $y = mx + b$ are said to be in *slope-intercept form*. Explain this title. The terminology refers to which of the two intercepts?

83. Drivers in distress on the Saw Mill River Parkway have two towing services to choose from: Brook's Body Shop charges \$3 per mile for the towing, and a fixed \$25 charge regardless of the length of the tow. Morgan Motors charges a flat \$5 per mile. On the same system of axes, represent each of these choices by a *linear* graph that plots the cost of the tow versus the length of the tow. If you needed to be towed, which service would you call, and why?

84. Driving from Boston to New York one day, Sasha covered the 250 miles in five hours. Because of heavy traffic, the 250-mile return took six hours and fifteen minutes. Calculate the average speeds for the trip *to* New York, the trip *from* New York, and the round trip. Explain why the terminology *average speed* is a bit misleading.

85. Find the value of x that makes $0.1x + 0.25(102 - x) = 17.10$ true.

86. So that it will be handy for paying tolls and parking meters, Lee puts pocket change (dimes and quarters only) into a cup attached to the dashboard. There are currently 102 coins in the cup, and their cash value is \$17.10. See if you can find a way to know how many of the coins are dimes and how many are quarters. Making a table can help. Where do you start with a guess? How do you zero in on an answer?

# of dimes	\$ from dimes	# of quarters	\$ from quarters	Total

87. Find all the values of x that make $0.1x + 0.25(102 - x) < 17.10$ a true statement.

88. Without using parentheses, write an expression equivalent to $3\{4(3x - 6) - 2(2x + 1)\}$.

89. One year after Robin deposits P dollars in a savings account that pays r % annual interest, how much money is in the account? Write an expression in terms of the variables P and r. If you can, write your answer using just a single P.

90. Morgan left home at 7:00 one morning setting off on a ten-mile bicycle trip. Soon thereafter, Morgan's parent realized that Morgan had forgotten a water bottle for the difficult journey. Morgan had a fifteen-minute head start, and was pedaling at 12 mph, while the parent pursued at 30 mph. Was Morgan able to get the water bottle before reaching the end of the journey that day? If so, where?

91. Farmer MacGregor needs to put a fence around a rectangular carrot patch that is one and a half times as long as it is wide. The project uses 110 feet of fencing. How wide is the garden?

92. Combine over a common denominator: $\frac{1}{a}+\frac{2}{3a}+3$

93. Graph the five points in the table all lie on a single line. Write an equation for the line and graph the line by hand. Without the graph, how would you confirm that these points were all on the same line?

x	y
-3	7
-2	5
-1	3
0	1
1	-1

94. If 6% of x is the same as 5% of 120, then what is x?

95. Find the solution sets and graph them on a number line. What do you notice about all of the sets put together?

(a) $46-3(x+10)=5x+20$

(b) $46-3(x+10)<5x+20$

(c) $46-3(x+10)>5x+20$

96. At 1 pm, you start out on your bike at 12 mph to meet a friend who lives 8 miles away. At the same time, the friend starts walking toward you at 4 mph. How much time will elapse before you meet your friend? How far will your friend have to walk?

97. The population of a small town increased by 25% two years ago and then decreased by 25% last year. The population is now 4500 persons. What was the population before the two changes?

98. Given that it costs \$2.75 less to buy a dozen doughnuts than to buy twelve single doughnuts, and that 65 doughnuts cost \$25.25, and that 65 = 5(12) + 5 , what is the price of a single doughnut?

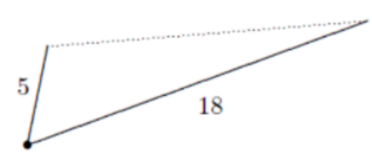
99. The volume of a circular cylinder is given by the formula $V = \pi r^2 h$.
    a. To the nearest tenth of a cubic cm, find the volume of a cylinder that has a 15-cm radius and is 12-cm high.
    b. Solve the volume formula for *h*. Then, if the volume is 1000 cc and the radius is 10 cm, find *h* to the nearest tenth of a cm.

100. Which of the following pairs of quantities vary directly?
    a. the circumference of a circle and the diameter of the circle
    b. the distance traveled in two hours and the (average) rate of travel
    c. the number of gallons of gasoline bought and the cost of the purchase
    d. the area of a circle and the radius of the circle

101. A jet, cruising at 26400 feet, begins its descent into Logan Airport, which is 96 miles away. Another jet, cruising at 31680 feet, is 120 miles from Logan when it begins its descent. Which of these two paths of descent is steeper? Explain.

102. There are infinitely many equations of lines of the form $ax + by = c$ but there are specific ones where *a, b* and *c* are consecutive integers. This is called a *family of lines* where the coefficients *a, b* and *c* are related in some way. The family of lines where a, b, and c are consecutive integers is an interesting group of lines. List at least 5 different lines in this family (both positive and negative coefficients) and graph them on your graph paper. What do you notice or wonder? Write down some conjectures and come to class with an argument why your conjecture might be true.

103. The diagram shows two steel rods hinged at one end. The other end is connected by a bungee cord (the dotted segment), whose unstretched length is 10 inches. The rods are 5 inches and 18 inches long. Use inequality symbols to describe all the possible lengths for the bungee cord, which stays being stretched.



104. According to the US Census Department, someone born in 1950 has a life expectancy of 68.2 years, while someone born in 1970 has a life expectancy of 70.8 years.
   a. What is a reasonable life expectancy of someone born in 1960?
   b. What is a reasonable life expectancy of someone born in 1980?
   c. What is a reasonable life expectancy of someone born in 2000?
   d. Part (a) is an *interpolation* question. Parts (b) and (c) are *extrapolation* questions. Which of your answers are you the most confident about? Explain.

105. Find all values of x that make $-2(x - 3) < 4$ true.

106. When it is 150 miles west of its destination, a jet is flying at 36920 feet. When it is 90 miles west of its destination, the jet is at 21320 feet. Using this data, sketch a graph of the jet's descent. Is a *linear model* reasonable to use in this situation? Explain.

107. Solve the following inequalities and shade their solution intervals on a number line.

$\frac{2x}{3}+\frac{3x+5}{2}\le 5$ **(b)** $\frac{1}{2}(x-1)+3>\frac{1}{3}(2x+1)-1$

108. A square game board is divided into smaller squares, which are colored red and black as on a checkerboard. All four corner squares are black. Let r and b stand for the numbers of red and black squares, respectively. What does the expression $b-r$ represent in the case? Can you find the actual value?

109. At noon, my odometer read 6852 miles. At 3:30 pm, it read 7034 miles.
   a. What was my average rate of change during these three and a half hours?
   b. Let t represent the number of hours I have been driving since noon and y represent my odometer reading. Write an equation that relates y and t. Assume constant speed.
   c. Graph your equation.
   d. Show that the point (5,7112) is on your line, and then interpret this point in the context of this problem.

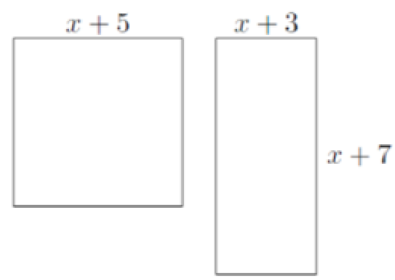
110. What is the slope between (3,7) and (5,4)? (5, 4) and (3,7)? (a, b) and (c,d)? (c,d) and (a,b)?

111. On top of a fixed monthly charge, Avery's cell phone company adds a fee for each text message sent. Avery's June bill was \$50.79, which covered 104 text messages. The bill for May, which covered 83 text messages, was only \$46.59.
   a. What is the price of a text message?
   b. What is the fixed monthly charge?
   c. What would Avery be charged for a month that included 200 text messages?
   d. What would Avery be charged for a month that included *m* text messages?

112. A friend suggested that I change my cell phone company. This new company has a fixed monthly charge of \$39.99, but it charges only 12 cents for each text message. Is this a better deal than the one described in the previous problem? Give evidence.

113. How far apart on a number line are **(a)** 12 and 18? **(b)** 12 and -7? **(c)** **-11** and -4?

114. For what values of *x* will the square and the rectangle shown at right have the same perimeter?



115. If you know that the point (3, 2) is on the line $y = 2x + b$. Find the value of *b*. Graph the line by hand on a set of axis that is suitable.

116. Are (2, 9) and (-3, -6) both on the line, $y = 4x + 6$. If not, find an equation for the line that does pass through both points. After you graph the line $y = 4x + 6$ find:
   a. the *y*-coordinate of the point on the line whose *x*-coordinate is 2;
   b. the *x*-coordinate of the point on the line whose *y*-coordinate is 2

117. The *absolute value* of a non-zero number can be defined by $|a| = -a$ or $|a| = a$ whichever is *positive.* Explain this statement, then evaluate each of the following:

(a) $|4|$ **(b)** $|-3|$ **(c)** $|5-8|$ **(d)** $|-3-1|$ **(e)** $|-5|-|12|$

118. In each of the following, describe the rate of change between the first pair and the second, assuming that the first coordinate is measured in minutes and the second coordinate is measured in feet. What are the units of your answer?

(a) (2, 8) and (5, 17) **(b)** (3.4, 6.8) and (7.2, 8.7) **(c)** $(\frac{3}{2}, \frac{-3}{4})$ and $(\frac{1}{2}, 2)$

119. If you double all the sides of a square, a larger square results. By what percentage has the perimeter increased? By what percentage has the area increased?

120. Combine the following fractions: $\dfrac{2}{3a}+\dfrac{1}{3}-\dfrac{4}{a}$

121. A toy manufacturer is going to produce a new toy car. Each one costs \$3 to make, and the company will also have to spend \$200 to set up the machinery to make them.
   a. What will it cost to produce the first hundred cars? the first n cars?
   b. The company sells the cars for \$4 each. Thus the company takes in \$400 by selling one hundred cars. How much money does the company take in by selling n cars?
   c. How many cars does the company need to make and sell in order to make a profit?

122. A cyclist rides 30 km at an average speed of 9 km/hr. At what rate must the cyclist cover the next 10 km in order to bring the overall average speed up to 10 km/hr.?

123. On a number line, what is the distance between 6 and 6? between 24 and 17? between 17 and 24? between t and 4? This last question is harder to answer because it depends on whether t is smaller than or greater than 4. Is the answer t - 4 or 4 - t ? This is an absolute value calculation: use absolute value signs to express the distance between t and 4. What is the distance between the numbers a and b on the number line? What is the relationship between $|p-q|$ and $|q-p|$?

124. Let $P=(x, y)$ and $Q=(1, 5)$. Using the formula that you know for slope, write an equation in terms of these coordinates that says that the slope of line PQ is 3. Show how this slope equation can be rewritten in the form $y-5=3(x-1)$. This linear equation is said to be in *point-slope form*. Explain the terminology. Find coordinates for three different points P that fit this equation.

125. (*Continuation*) What do the lines $y=3(x-1)+5$, $y=-2(x-1)+5$, and $y=1/2(x-1)+5$ all have in common? How do they differ from each other?

126. Given that $48 \le n \le 1296$ and $24 \le d \le 36$, what are the largest and smallest values that the expression $\frac{n}{d}$ could possibly have? Write your answer in the form

127. Write an equation for the line that goes through the point (1, 5) and that has slope $\frac{2}{3}$.

128. Consider the family of lines $ax + by = c$, where a, b and c are consecutive even numbers. Write some of the equations that are in this family of lines. Make sure that some of the coefficients are negative as well as positive, increasing as well as decreasing to get many different types of lines. After you graph the lines what do you notice or wonder? Can you prove your conjectures?

129. Write an equation for the line that goes through the point (1, 5) and that has slope $\frac{2}{3}$.

130. The equation $5x - 8y = 20$ expresses a linear relationship between x and y. The point (15, 7) is either on the graph of this line, above it, or below it. Which? How do you know?

131. The table at right shows data that Morgan collected during a 10-mile bike ride that took 50 minutes. The cumulative distance (measured in miles) is tabled at ten-minute intervals.

a. Using Desmos, make a *scatter plot* of this data. Why might you expect the data points to line up? Why do they not line up?

b. Morgan's next bike ride lasted for 90 minutes. Estimate its length (in miles), and explain your method. What if the bike ride had lasted t minutes; what would its length be, in miles?

time (mins)	distance (mi)
0.0	0.0
10	2.3
20	4.4
30	5.7
40	8.2
50	10.0

132. Write an equation for the line that contains the points in the table, and make up a context for it.

x	0	15	30	45	60
y	100	160	220	280	340

133. On a number line, how far is each of the following numbers from zero?

(a) 45 **(b)** -7 **(c)** x **(d)** 0

134. Solve: **(a)** $A = \frac{1}{2}bh$ for b; **(b)** $A = 2\pi rh + 2\pi r^2$ for h.

135. On a number line, how far is each of the following numbers from 5?

(a) 17 **(b)** 4 **(c)** x

136. Solve the equation $Q - R = m(C - D)$ for m. In other words, what does m equal in terms of the other letters in this equation? Does this representation tell you how to get m if you were given two other representations in the form (C, Q) and (D, R)? What is m?

137. Write the equation of the line that goes through the point (-3, 4) and:
a. has slope equal to 5 b. is horizontal c. is vertical

138. When asked to find the distance between two numbers on a number line, Jamie responded with the following answers. In each case, what two numbers do you think Jamie was talking about?

(a) $|9-4|$ **(b)** $|9+4|$ **(c)** $|x-7|$ **(d)** $|3-x|$ **(e)** $|x+5|$ **(f)** $|x|$

139. Find an equation for the line containing the points (-3, 0) and (0, 4). Be ready to justify the method you used. Is there a more efficient way?

140. To graph linear equations such as $3x + 5y = 30$, one can put the equation into slope-intercept form, but (unless the slope is needed) it is easier to find the x- and y-intercepts and use them to sketch the graph. Find the axis intercepts of each of the following and use them to draw the given line. An equation $ax + by = c$ is said to be in *standard form*.

(a) $20x + 50y = 1000$ **(b)** $4x - 3y = 72$

141. Write an equation in point-slope form for
   a. the line that goes through (2, 5) and (6, 3);
   b. the line that goes through point (h, k) and that has slope m.

142. *Definition of a function*: Equations such as $y = 2x + 5$ and $h = 5t - 3$ can also be written as *functions.* Look in the reference section and see what that means. Write in your own words what you think a function is and rewrite these two equations in *function notation*.

143. Casey goes for a bike ride from Avenues School to the Bronx Zoo, while an odometer keeps a cumulative record of the number of miles traveled. The equation $m = 12t + 37$ describes the odometer reading m after t hours of riding. What is the meaning of 12 and 37 in the context of this trip?

144. Find an equation for the line that passes through the points (4.1, 3.2) and (2.3, 1.6).

145. Find values for all the points on a number line that are
(a) 6 units from 0; **(b)** 6 units from 6; **(c)** 6 units from -7; **(d)** 6 units from x.

146. Rearrange the eight words "between", "4", "the", "17", "is", "and", "x", and "distance" to form a sentence that is equivalent to the equation $|x-17|=4$. By working with a number line, find the values of x that fit the equation.

147. Write the equation of the line that goes through the points $A(-2, 5)$ and $B(3, 6)$ in point/slope form.

148. A recent CNN poll about crime in schools reported that 67% of Americans approved of a bill being debated in Congress. The CNN report acknowledged a 3% margin of error.
   a. Make a number-line graph of the range of approval ratings in this report.
   b. Explain why the range of approval ratings can be described by $|x-0.67| \le 0.03$

149. Translate the sentence "the distance between x and 12 is 20" into an equation using algebraic symbols. What are the values of x being described?

150. The solution of $|x| = 6$ consists of the points 6 and -6. Show how to use a test point on the number line to solve and graph the inequality $|x| \le 6$. Do the same for $|x| \ge 6$.

151. As you know, temperatures can be measured by either Celsius or Fahrenheit units; $30^\circ C$ is equivalent to $86^\circ F$, $5^\circ C$ is equivalent to $41^\circ F$, and $10^\circ C$ 10 is equivalent to $14^\circ F$. Plot this data with C on the horizontal axis and F on the vertical axis.
   a. Verify that these three data points are *collinear* .
   b. Find a linear equation that relates C and F.
   c. Graph F versus C. In other words, graph the linear equation you just found.
   d. Graph C versus F . You will need to replot the data, with C on the vertical axis.
   e. Water boils at 212 °F and freezes at 32 °F at sea level. Find the corresponding Celsius temperatures.
   f. Is it ever the case that the temperature in degrees Fahrenheit is the same as the temperature in degrees Celsius?

152. Translate "x is 12 units from 20" into an equation. What are the values of x being described?

153. On a number line, graph $|x| < 2$. Now graph $|x-5| < 2$. How does the second interval relate to the first interval?

154. Translate the sentence "x and y are twelve units apart" into an algebraic expression. Find a pair (x, y) that fits this description. How many pairs are there?

155. Write the equation of the line that is parallel to $y = 2x + 6$ but goes through (-2,-3).

156. The equation $|x - 7| = 2$ is a translation of "the distance from x to 7 is 2." Translate $|x - 7| \leq 2$ into English, and graph its solutions on a number line.

157. Convert "the distance from -5 to x is at most 3" into symbolic form, and solve it.

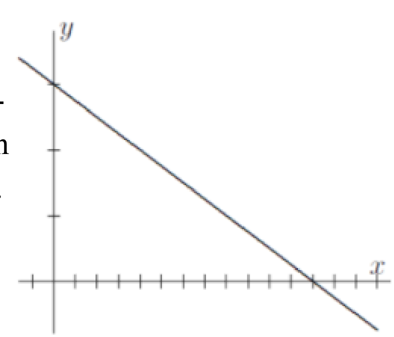
158. In class, Evan read $-75 < 2$ as "negative 75 is less than 2." Neva responded by saying "I'm thinking that 75 is a larger number than 2." How would you resolve this apparent conflict?

159. Verify that (0, 4) is on the line $3x + 2y = 8$. Find another point on this line. Use these points to calculate the slope of the line. Is there another way to find the slope of the line?

160. Graph a horizontal line through the point (3, 5). Choose another point on this line. What is the slope of this line? What is the y-intercept of this line? What is an equation for this line? Describe a context that could be modeled by this line.

161. Graph a vertical line through the point (3, 5). Does this line have a slope or y-intercept? What is an equation for this line? Describe a context that could be modeled by this line.

162. After successfully solving an absolute-value problem, Ariel spilled Heath Bar Crunch© all over the problem. All that can be read now is, "The distance between x and (mess of ice cream) is (another mess of ice cream)." Given that Ariel's answers are $x = -3$ and $x = 7$, reconstruct the missing parts of the problem.



163. The figure shows the graph of $20x + 40y = 1200$. Find the x- and y-intercepts, the slope of the line, and the distances between tick marks on the axes. Duplicate this figure on a graphing tool. What window settings did you use?

164. Make a general statement about the slopes of horizontal lines and vertical lines.

165. The average of three different positive integers is 8. What is the largest integer that could be one of them?

166. Is the point (8.4, 23) below, on, or above the line $3x - y = 2$? Justify your answer numerically first and then in some other way.

167. A handicapped-access ramp starts at ground level and rises 27 inches over a distance of 30 feet. What is the slope of this ramp?

168. Jay thinks that the inequality $k < 3$ implies the inequality $k^2 < 9$, but Val thinks otherwise. Who is right, and why?

169. The specifications for creating a piece of metal with an industrial machine state that it must be 12 cm long, within a 0.01-cm tolerance. What is the longest the piece is allowed to be? What is the shortest? Using l to represent the length of the finished piece of metal, write an absolute-value inequality that states these conditions.

170. A movie theater charges \$6 for each adult and \$3 for each child. If the total amount in ticket earnings one evening was \$1428 and if there were 56 more children than adults, then how many children attended?

171. If $|x + 1| = 5$, then $x + 1$ can have two possible values, 5 and -5. This leads to two equations $x + 1 = 5$ and $x + 1 = -5$. If $|2x - 7| = 5$, what possible values could the expression $2x - 7$ have? Write two equations using the expression $2x - 7$ and solve them.

172. Write two equations without absolute value symbols that, in combination, are equivalent to $|3x + 5| = 12$. Solve each of these two equations.

173. Given that $0.0001 \leq n \leq 0.01$ and $0.001 \leq d \leq 0.1$, what are the largest and smallest values that $\frac{n}{d}$ can possibly have? Write your answer *smallest* $\leq \frac{n}{d} \leq$ *largest*.

174. A *lattice point* is defined as a point whose coordinates are integers. If (3, 5) and (2, 1) are two points on a line, find three other lattice points on the same line.

175. The equation $13x + 8y = 128$ expresses a linear relationship between x and y. The point (5, 8) is on, or above, or below the linear graph. Which is it? How do you know?

176. Show that the equation $y=\frac{7}{3}x-\frac{11}{8}$ can be rewritten in the form $ax + by = c$ where a, b and c are integers. What are those integers?

177. Fill in the blanks:

a. The inequality $|x-1.96|<1.04$ is equivalent to "x is between______ and_____ ."

b. The inequality $|x-2.45|\geq 4.50$ is equivalent to "x is not between______ and_____ ."

178. Find the value for h for which the slope of the line through $(-5,6)$ and $(h,12)$ is 3/4. It might help to draw a picture or think of a formula.

179. Solve the equation $0.05x + 0.25(30 - x) = 4.90$. Invent a context for the equation.

180. The data in each table fits a direct variation. Complete each table, write an equation to model its data, and sketch a graph

(a)

x	2		5	8
y	5	10		20

(b)

x	2	4	6	
y	3	6		18

181. For each of the following equations, find the x-intercept and y-intercept. Then use them to calculate the slope of the line.

(a) $3x + y = 6$ **(b)** $x - 2y = 10$ **(c)** $4x - 5y = 20$ **(d)** $ax + by = c$

182. Blair's average on the first five in-class tests is 67. If this is not pulled up to at least a 70, Blair will not be allowed to watch any more of their favorite Netflix shows.. To avoid losing those access privileges, what is the lowest score Blair can afford to earn on the last in-class test? Assume that all tests carry equal weight.

183. Sketch the graphs of $y = 2x$, $y = 2x + 1$, and $y = 2x + 2$ all on the same coordinate-axis system. Find the slope of each line. How are the positions of the lines related to one another?

184. I have 120 cm of framing material to make a picture frame, which will be most pleasing to the eye if its height is 2/3 of its width. What dimensions should I use?

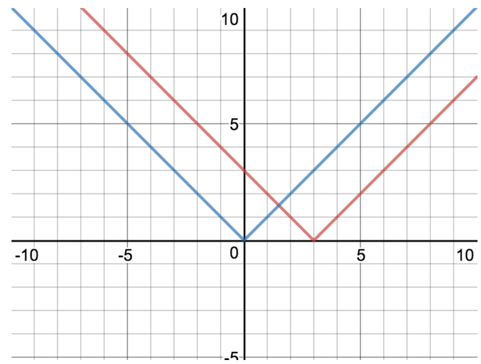
185. Describe the relationship between the following pairs of numbers:

(a) 24 - 11 and 11 - 24 **(b)** $x - 7$ and $7 - x$ **(c)** $|x-7|$ and $|7-x|$

186. In each case, decide whether the three points given are collinear:
(a) (-4, 8), (0, 2), and (2, -1) **(b)** (350, 125), (500, 300), and (650, 550)

187. Use Desmos to graph $y = |x+5|$ and $y = |x-3|$, then describe in general terms how the graph of $y = |x|$ is transformed to produce the graphs of $y = |x-h|$.

188. Write an equation for each of the absolute value graphs shown at the right.

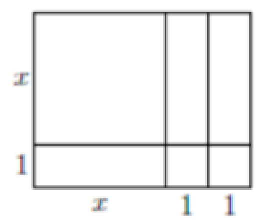


189. Solve the equation $C = \frac{5}{9}(F-32)$ for F.

190. Using graph paper, draw the line through the point (0, 6) whose slope is 2/3. If you move 24 units to the right of (0, 6), and then move up to the line, what is the *y*-coordinate of the point you reach?

191. (Continuation) Find an equation for the line. What is the *x*- intercept of the line?

192. Using distribution, write the multiplication problem $(x+1)(x+2)$ without parentheses. Explain how the diagram at right illustrates this product.



193. Sketch on the same axes the graphs of $y = |x|$ and $y = |x| - 2$. Label the x and *y*-intercepts. Name the ways in which the two graphs are similar and different?

194. The manager at Jen and Berry's Ice Cream Company estimates that the cost C (in dollars) of producing n quarts of ice cream in a given week is given by the equation $C = 560 + 1.20n$.

a. During one week, the total cost of making ice cream was \$1070. How many quarts were made that week?

b. Write a sentence explaining the meanings of the "560" and the "1.20" in the cost equation.

195. Draw a line through the origin with a slope of 0.4. Draw a line through the point (1, 2) with a slope of 0.4. How are these two lines related? What is the vertical distance between the two lines? Find an equation for each line.

196. As anyone knows who has hiked up a mountain, the higher you go, the cooler the temperature gets. At noon on July 4th last summer, the temperature at the top of Mt. Washington, elevation 6288 feet, was 56 °F. The temperature at base camp in Pinkham Notch, elevation 2041 feet, was 87 °F. It was a clear, still day. At that moment, a group of hikers reached Tuckerman Junction, elevation 5376 feet. To the nearest degree, calculate the temperature the hikers were experiencing at that time and place. When you decide how to model this situation, what assumptions did you make?

197. Graph $y = |x| + 3$ and $y = |x| - 5$, then describe in general terms how the graph of $y = |x|$ is transformed to produce the graph of $y = |x| + k$. How can you tell from the graph whether k is positive or negative?

198. Randy phones Sandy about a homework question, and asks, "The vertex of the graph of *y* equals the absolute value of *x* plus four is (-4, 0), isn't it?" Sandy answers, "No, the vertex is (0, 4)." Who is right? Explain your thinking.

199. Solve $\frac{3m}{4} + \frac{3}{8} = \frac{m}{3} - \frac{5}{6}$ for *m* as a fraction in lowest terms.

200. Find two different ways of determining the slope of the line $11x + 8y = 176$.

201. Find the *x*- and *y*-intercepts of $y = |x - 3| + 5$, find the coordinates of its vertex, and then sketch the graph of this equation.

202. When weights are placed on the end of a spring, the spring stretches. If a three-pound weight stretches the spring to a length of 4.25 inches, a five-pound weight stretches the spring to a length of 5.75 inches, and a nine-pound weight stretches the spring to a length of 8.75 inches, what was the initial length of the spring?

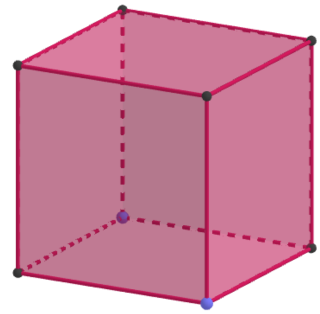
203. Given that *y* varies directly with *x* and y = 60 when *x* = 20, find *y* when *x* = 12.

204. Solve for *x*: $\frac{1}{2}(x-2) + \frac{1}{3}(x-3) + \frac{1}{4}(x-4) = 10$

205. Apply the distributive property to write without parentheses and collect like terms:

a. **(a)** $x(x-3)+2(x-3)$ **(b)** $2x(x-4)-3(x-4)$ **(c)** $x(x-2)+2(x-2)$

206. A cube measures x cm on each edge.



a. Find a formula in terms of x for the volume of this cube in cubic centimeters (cc).
b. Evaluate this formula when $x = 1.5$ cm; when $x = 10$ cm.
c. Write an expression for the area of one of the faces of the cube. Write a formula for the total surface area of all six faces.
d. Evaluate this formula when $x = 1.5$ cm; when $x = 10$ cm.
e. Although area is measured in square units and volume in cubic units, is there any cube for which the number of square units in the area of its faces would equal the number of cubic units in the volume?

207. The amount of fuel a car uses in miles per gallon is called its *fuel efficiency*, how good it is a conserving fuel. The fuel efficiency of a car depends on the speed at which it is driven. For example, consider Kit's Volvo. When it is driven at r miles per hour, it gets $m = 32 - 0.2\,|r - 55|$ miles per gallon. Graph m versus r, for $0 < r < 80$. Notice that this graph has a vertex. What are its coordinates?

208. (*Continuation*) Solve the inequality $30 \le 32 - 0.2\,|r - 55|$ and express the solution interval graphically. What is the meaning of these r-values to Kit.

209. Asked to solve the inequality $3 < |x - 5|$ at the board, Corey wrote "$8 < x < 2$," Sasha wrote "$x < 2$ or $8 < x$," and Avery wrote "$x < 2$ and $8 < x$." What do you think of these answers? Do any of them agree with your answer?

210. On graph paper, think of the equation $y = |x + 4| - 2$ as a transformation of $y = |x|$. How has it been moved? Sketch both graphs by hand.

211. Apply the distributive property to write without parentheses and collect like terms:

(a) $(x+2)(x-3)$ **(b)** $(2x-3)(x-4)$ **(c)** $(x+2)(x-2)$

212. If the width and length of a rectangle are both increased by 10%, by what percent does the area of the rectangle increase? By what percent does the perimeter of the rectangle increase?

213. The upward velocity of the water in a particular fountain is given in meters per second by $v = -32t + 44$, where t is the number of seconds after the water leaves the fountain. While going upward the water slows down until, at the top, the water has a velocity of zero. How long does it take each water particle to reach its maximum height?

214. Compare the graphs of $y = x - 3$ and $y = |x - 3|$. How are they related?

215. Morgan's way to solve the equation $|2x - 7| = 5$ is to first write $|x - 3.5| = 2.5$. Explain this approach, and then finish the job.

216. A train is leaving in 11 minutes and you are one mile from the station. Assuming you can walk at 4 mph and run at 8 mph, how much time can you afford to walk before you must begin to run in order to catch the train?

217. A friend told Sandy that "absolute value makes everything positive." So, Sandy rewrote the equation $|x - 6| = 5$ as $x + 6 = 5$. Do you agree with the statement, or with what Sandy did to the equation? Explain your answer.

218. For each of the following points, find the distance to the y-axis. Think of distance as "the shortest distance":

(a) (11, 7) **(b)** (-5, 9) **(c)** (4, y) **(d)** (x, -8)

219. Given the line $y = \frac{1}{2}x + 6$, write an equation for the line through the origin that has the same slope. Write an equation for the line through (2, -4) that has the same slope.

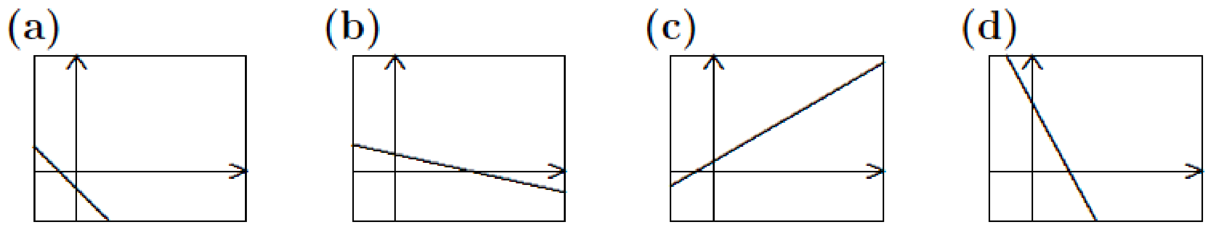
220. Rewrite the English sentence that describes all of the points that are "7 units away from -3" as an absolute value statement mathematically. Be prepared to describe how you arrived at your answer.

221. The table shows the population of New Hampshire at the start of each of the last six decades.

Year	*Population*
1960	606,921
1970	746,284
1980	920,610
1990	1,113,915
2000	1,238,415
2010	1,316,472

a. Write an equation for the line that contains the data points for 1960 and 2010.
b. Write an equation for the line that contains the data points for 2000 and 2010.
c. Make a scatter plot of the data. Graph both lines on it.
d. Use each of these equations to predict the population of New Hampshire at the beginning of 2020. Explain why these lines might provide an accurate forecast.

222. Which of the following screens could represent the graph of $9x + 5y = 40$?



223. For each of these absolute-value equations, write two equations without absolute-value symbols that are equivalent to the original. Solve each of the equations.

(a) $2|x + 7| = 12$ **(b)** $3 + |2x + 5| = 17$ **(c)** $6 - |x + 2| = 3$ **(d)** $-2|4 - 3x| = -14$

224. Hearing Yuri say "This line has no slope," Tyler responds "Well, 'no slope' actually means slope 0." What are they talking about? Do you agree with either of them?

225. Suppose a flat, rectangular board is built by gluing together a number of square pieces of the same size.

a. If 20 squares are glued together to make a 4 by 5 rectangular board, how many of these squares are completely surrounded by other squares?

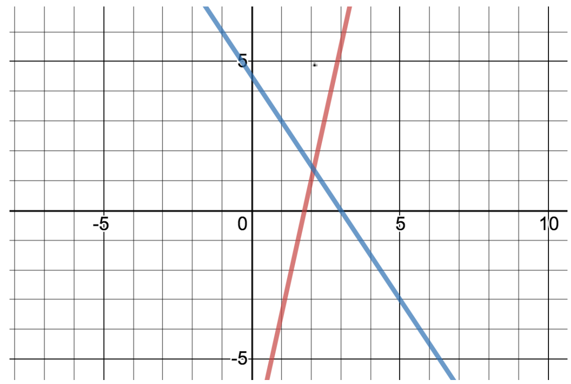
b. If the dimensions of the finished rectangular board are m by n, how many squares (in terms of n and m) are completely surrounded by other squares?

226. The edges of a solid cube are $3p$ cm long. At one corner of the cube, a small cube is cut away. All its edges are p cm long. In terms of p, what is the total surface area of the remaining solid? What is the volume of the remaining solid? Make a sketch.

227. Lee's pocket change consists of x quarters and y dimes. Put a dot on every lattice point (x, y) that signifies that Lee has exactly one dollar of pocket change. What equation describes the line that passes through these points? Why doesn't it make sense to connect the dots in the context of this problem? x and y in this problem are examples of *discrete variables*, variables that represent only *integer values*.

228. (*Continuation*) Put a dot on every lattice point (x, y) that signifies that Lee has at most one dollar in pocket change. How many such dots are there? What is the relationship between Lee's change situation and the inequality $0.25x + 0.10y \le 1.00$.

229. The figure shows the graphs of two lines. By looking at the axes, (the axis markings are one unit apart) estimate the coordinates of the point that belongs on both lines (where the lines cross).



230. (*Continuation*) The *system of equations* that has been graphed is $\begin{cases} 9x - 2y = 16 \\ 3x + 2y = 9 \end{cases}$. Jess took one look at the equation and knew right away what to do, "Just add the two equations together and you will find out quickly what x is." Follow this advice and then explain why it works.

231. (*Continuation*) Now that you have the x-coordinate of the point of intersection, what should you do to get the y-coordinate? Does it matter which original equation you use?. Do the coordinates of the *point of intersection* agree with your estimate? These coordinates are called a *simultaneous solution* of the original system of equations. What do you think a simultaneous solution is?

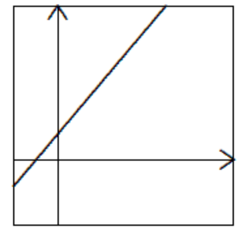
232. Consider the line with equation $y = 2(x+3) - 1$. Write an equation for the line which has the same slope and contains the point (3, -1).

233. State at least 4 equations in the family of lines $ax + by = c$, where a, b and c, differ by a value of 5. In other words, $b - a = 5$ and $c - b = 5$. Make sure that they are actually different lines. Graph your equations on the coordinate axes. What do you notice or wonder? Find a way to prove any conjectures that you discover.

234. In 1990 a company had a profit of \$420000. In 1995 it reported a profit of \$1400000. Find the average rate of change of its profit for that period, expressed in dollars per year.

235. Most linear equations can be rewritten in slope-intercept form $y = mx + b$. Give an example that shows that not all linear equations can be written in slope-intercept form.

236. Which of the following could be the equation whose graph is shown at right? To support your answer, explain what portion of the x-axis and y-axis are shown?

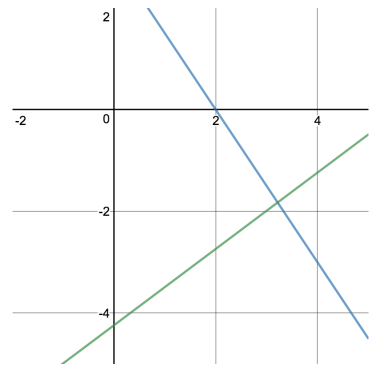


(a) $3y-7x=28$ **(b)** $x+2y=5$

(c) $12x=y+13$ **(d)** $y-0.01x=2000$

237. Find the values of x and y that fit both the equation $2x - 3y = 8$ and $4x + 3y = -2$. Be ready to support your method.

238. The figure at right shows the graphs of two lines . First use the figure to estimate the coordinates of the point that belongs on both lines. The system of equations is $\begin{cases} 3x+2y=6 \\ 3x-4y=17 \end{cases}$



239. Randy took one look at these equations and knew right away what to do. “Just subtract the equations and you will find out quickly what y is.” Follow this advice.

240. (*Continuation*) How would you find the missing y-value of the point of intersection now that you have the y-value?. Compare the intersection coordinates with your estimate.

241. (*Continuation*) If you *add* the two given equations, you obtain the equation of yet another line. Add its graph to the figure. You should notice something. Was it expected?

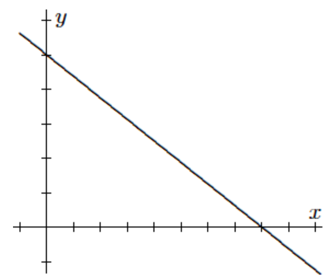
242. Brett is holding three quarters and five dimes. Does Brett have more than one dollar or less than one dollar? Does the point (3, 5) lie above or below the line $0.25x + 0.10y = 1.00$? How does the first question asked relate to the second?

243. Find the value of x that fits the equation $\frac{1}{2}x+\frac{1}{3}x+\frac{1}{4}x=26$.

244. A hot-air balloon ride has been set up so that a paying customer is carried straight up at 50 feet per minute for ten minutes and then immediately brought back to the ground at the same rate. The whole ride lasts twenty minutes. Let h be the height of the balloon (in feet) and t be the number of minutes since the ride began. Draw a graph of h versus t. What are the coordinates of the vertex? Find an equation that expresses h in terms of t.

245. Fitness Universe has a membership fee of \$50, after which individual visits to the gym are \$5.50. Non-members pay \$8.00 per visit. Stuart is going to exercise at the gym regularly, and is wondering whether it makes sense to become a member. How regularly would Stuart need to visit this gym, in order for a membership to be worth it?

246. What is the slope of the line graphed at the right, if the distance between the x-tick marks is 2 units and the distance between the y-tick marks is 1 unit? the distance between the x-tick marks is 100 units and the distance between the y-tick marks is 5 units?



247. My sleeping bag is advertised to be suitable for temperatures T between 20 degrees below zero and 20 degrees above zero (Celsius). Write an absolute-value inequality that describes these temperatures T.

248. Pat has x quarters and y dimes, and, in addition, has no more than two dollars. Write several inequalities that represent this situation and then graph all points in the coordinate plane that satisfy this condition.

249. Graph the equation $2x + 3y = 6$. Now graph the inequality $2x + 3y \le 6$ by shading all points (x, y) that fit it. Notice that this means shading all the points on one side of the line you drew. Which side? Use a test point like $(0, 0)$ to decide.

250. Some questions about the line that passes through the points $(-3, 2)$ and $(5, 6)$:
   a. Find the slope of the line.
   b. Is the point $(10, 12)$ on the line? Justify your answer.
   c. Find y so that the point $(7, y)$ is on the line.

251. Graph $y = |x - 1|$. Use your graph to find all values of x that satisfy $|x - 1| \le 3$.

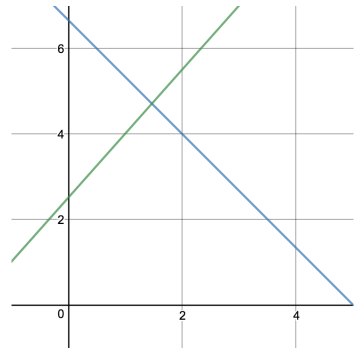
252. Find values for x and y that fit both of the equations $5x + 3y = 8$ and $4x + 3y = -2$.

253. Casey can peel k apples in 10 minutes.
   a. In terms of k, how many apples can Casey peel in one minute?
   b. How many apples can Casey peel in m minutes?
   c. In terms of k, how many minutes does it take Casey to peel one apple?
   d. How many minutes does it take Casey to peel p apples?

254. Express each as a single fraction: **(a)** $\frac{1}{a}+\frac{2}{b}+\frac{3}{c}$ **(b)** $\frac{1}{a}+\frac{1}{b+c}$ **(c)** $1+\frac{1}{b+c}$

255. Graph $y = 3|x - 2| - 6$, and find coordinates for the vertex and the x- and y-intercepts.

256. The figure at right shows the graphs of two lines. Use the figure to estimate the coordinates of the point that belongs to both lines. The system of equations is $\begin{cases} 4x+3y=20 \\ 3x-2y=-5 \end{cases}$. Lee took one look at the equations and announced a plan: "Just multiply the first equation by 2 and the second equation by 3." What does changing the equations in this way do to their graphs?



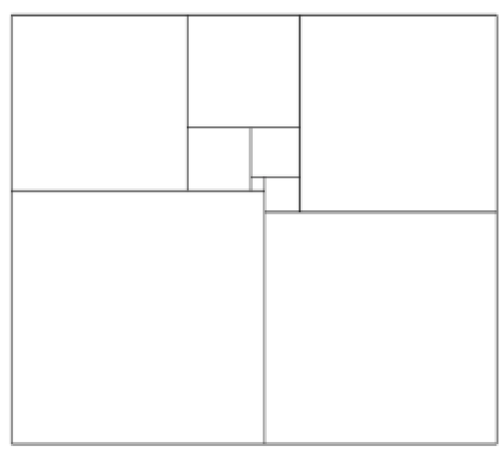
257. (*Continuation*) Lee's plan has now created a familiar situation. Do you recognize it? Complete the solution to the system of equations. Do the coordinates of the point of intersection agree with your initial estimate?

258. Sandy's first four test scores this term are 73, 87, 81 and 76. To have at least a B test grade, Sandy needs to average at least 80 on the five term tests (which count equally). Let t represent Sandy's score on the fifth test, and write an inequality that describes the range of t-values that will meet Sandy's goal.

259. Graph solutions on a number line:

(a) $|x + 8| < 20$ **(b)** $|2x - 5| \le 7$ **(c)** $3|4 - x| \ge 12$

260. Plot some points (x, y) on the coordinate plane where the x-coordinate is greater than the y-coordinate. Where are all such points? Shade the points in the plane whose x-coordinates are greater than their y-coordinates. Write an inequality that describes these points.



261. The diagram at right shows a rectangle that has been cut into nine square pieces, no two being the same size. Given that the smallest piece is 2 cm by 2 cm, figure out the sizes of the other eight pieces. A good strategy is to start by guessing the size of one of the pieces adjacent to the smallest piece. By checking your guess, you will discover the hidden equation.

262. Solve the system of equations $2x + y = 5$ and $5x - 2y = 8$ algebraically. Check your answer graphically.

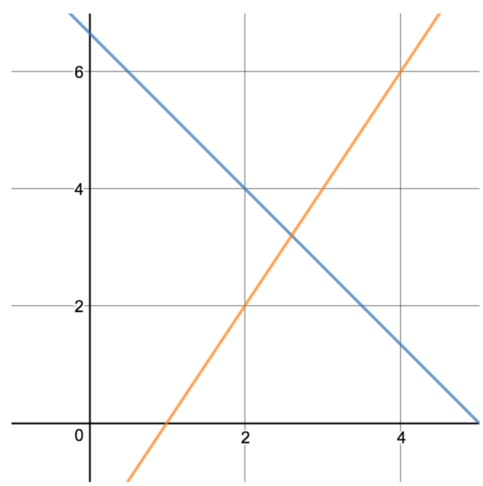
263. A large telephone company sent out an offer for pre-paid phone cards. The table below accompanied the ad and summarized their offer. Does this data form a linear relationship? Explain your answer. Which offer has the best rate per minute?

75-minute card	150-minute card	300-minute card	500-minute card	1000-minute card	1500-minute card
\$4.95	\$9.90	\$19.80	\$30.00	\$56.00	\$75.00

264. Find an equation for each of the following lines. When possible, express your answer in both point-slope form and slope-intercept form.
   a. The line passes through (3, 5), and has -1.5 as its slope.
   b. The line is parallel to the line through (-8, 7) and (-3, 1), and has 6 as its x-intercept.
   c. The line is parallel to the line x = -4, and it passes through (4, 7).

265. The figure at right shows the graphs of two lines. Use the figure to estimate the coordinates of the point that belongs to both lines. The system of equations is

$$\begin{cases} 4x + 3y = 20 \\ y = 2x - 2 \end{cases}$$



Think about a way that you can use the second equation to find an expression for the variable y, to substitute for the y in the first equation. Substitute and then solve for x. How could you then find the value of y?

266. A certain rectangle has length $2x + 1$ and width $x + 1$, find an expression for its area. Your answer should be in the form of a *quadratic expression*. Why do you think it's called this?

267. Farmer MacGregor wants to know how many cows and ducks are in the meadow. After counting 56 legs and 17 heads, the farmer knows. Consider the following table:

# of cows	# of ducks	Legs	Heads
1	1		
1	2		
2	1		
c	d		

a. Fill in the values for Legs and Heads for the given number of cows and ducks
b. Write an equation for the number of Legs on c cows and d ducks.
c. Write an equation for the number of Heads on c cow and d ducks.
d. Use the information that Farmer MacGregor counted to solve for the number of cows and ducks there are in the meadow.

268. You have learned how to *factor* numbers that were not prime. Factor the number 24,480 into its prime factors. What if you had something like the expression 7(24,480 + 2,125)? Can you find the *largest common factor* of 24,480 and 2,125 and "*factor it out*" of the sum inside the parenthesis? Rewrite 7(24,480 + 2,125) as a single number outside the parentheses multiplied by a different sum inside the parentheses.

269. Three gears are connected so that two turns of the first wheel turn the second wheel nine times and three turns of the second wheel turn the third wheel five times.
a. If you turn the first wheel once, how many times does the third wheel turn?
b. How many times must you turn the first wheel so that the third wheel turns 30 times?

270. Find the point (x, y) that fits both of the equations $y = 1.5x + 2$ and $9x + 4y = 41$.

271. Sam boards a ski lift, and rides up the mountain at 6 miles per hour. Once at the top, Sam immediately begins skiing down the mountain, averaging 54 miles per hour, and does not stop until reaching the entrance to the lift. The whole trip, up and down, takes 40 minutes. Assuming the trips up and down cover the same distance, how many miles long is the trip down the mountain?

272. If the price of a stock goes from \$4.25 per share to \$6.50 per share, by what percent has the value of the stock increased?

273. What set of points does this absolute value statement, $|x-6|<3$, describe on a number line? Write your answer as an English sentence.

274. Your company makes spindles for the space shuttle. NASA specifies that the length of a spindle must be 12.45 ± 0.01 cm. What does this mean? What are the smallest and largest acceptable lengths for these spindles? Write this range of values as an inequality, letting L stand for the length of the spindle. Write another inequality using absolute values that models these constraints.

275. *Factor* each of the following quadratic expressions. Remember to look for the greatest common factor and factor it out of the sum or difference:
(a) $x^2 - 4x$ **(b)** $2x^2 - 6x$ **(c)** $3x^2 - 15x$ **(d)** $-2x^2 - 7x$

276. Alex and Tracy are having a conversation about variables one day. Alex says, "If $x \cdot y = 0$ then one of the numbers must be zero or maybe both of them!." Tracy responds by saying, "No, if $x \cdot y = 0$, then $x = 0$, because then the product will always be zero." Does either student have the right argument? Can you think of numerical examples that verify either statement?

277. (*Continuation*) The *zero-product property* says that $a \cdot b = 0$ is true if $a = 0$ or $b = 0$ is true, and *only* if $a = 0$ or $b = 0$ is true. Explain this property in your own words (looking up the word *or* in the Reference section if necessary). Apply it to solve these *quadratic equations*:
(a) $x^2 - 4x = 0$ **(b)** $2x^2 - 6x = 0$ **(c)** $3x^2 - 15x = 0$ **(d)** $-2x^2 - 7x = 0$

278. Use the distributive property to multiply $(x+p)(x+q)$. The result of this multiplication can be expressed in the form $x^2 + ■x + \triangle$ what do ■ and Δ stand for? Can you justify your answer with a geometric representation where $x+p$ and $x+q$ are the length and width of a rectangle?

279. (*Continuation*) When attempting to factor $x^2 + 5x + 4$ into a product of two *binomials* of the form $(x + p)(x + q)$, Dylan set up the *identity* $x^2 + 5x + 4 = (x + \_\_)(x + \_\_)$. Using a *trial-and-error* process, try to figure out what numbers go in the blank spaces. What is the connection between the numbers in the blank spaces and the coefficients 5 and 4 in the quadratic expression being factored?

280. Verify that it is true that $23^2 - 20^2 = (23-20)(23+20)$ and explain why. Use this idea to easily compute $31^2 - 29^2$.

281. What is unusual about the graphs of the equations $9x - 12y = 27$ and $-3x + 4y = -9$?

282. Now consider the family of lines $ax + by = c$ such that *a, b* and *c* are consecutive multiples of each other. What would that mean? Give some examples of such lines in this family and graph them. What do you notice or wonder?

283. The number of miles a vehicle gets per gallon is called its fuel efficiency. The fuel efficiency, *m* (in miles per gallon) of a truck depends on the speed *r* (in miles per hour) at which it is driven. The relationship between *m* and *r* usually takes the form $m = a|x-h| + k$. For Sasha's truck, the best fuel efficiency is 24 miles per gallon, attained when the truck is driven at 50 miles per hour. When Sasha drives at 60 miles per hour, however, the fuel efficiency drops to only 20 miles per gallon.

r	*m*
60	20
50	24
40	
30	
20	
10	

a. Explain why the expression for fuel efficiency, *m*, might be explained by an absolute value function.
b. Find another driving speed *r* for which the fuel efficiency of Sasha's truck is exactly 20 mpg.
c. Fill in the rest of the missing entries in the table.
d. Draw a graph of *m* versus *r*, for $0 < r \leq 80$.
e. Find the values of *k*, *a*, and *h*.

284. With parental assistance, Corey buys some snowboarding equipment for $500, promising to pay $12 a week from part-time earnings until the 500-dollar debt is retired. How many weeks will it take until the outstanding debt is under $100? Write an inequality that models this situation and then solve it algebraically.

285. Factor each expression:

(a) $3x^2 + 12x$ **(b)** $x^2 + 8x + 15$ **(c)** $4xy + 2y^2$

286. On 3 January 2004, after a journey of 300 million miles, the rover Spirit landed on Mars and began sending back information to Earth. It landed only six miles from its target. This accuracy is comparable to shooting an arrow at a target fifty feet away and missing the exact center by what distance?

287. Graph $y = 2|x+1| - 3$, then describe in general terms how the graph of $y = |x|$ is transformed to produce the graph of $y = a|x - h| + k$.

288. Find an equation for the line that passes through the point (-3, 6), parallel to the line through the points (0, -7) and (4, -15). Write your answer in point-slope form.

289. I recently paid \$85.28 for 12.2 pounds of coffee beans. What was the price per pound of the coffee? How many pounds did I buy per dollar?

290. Sid has a job at Morgan Motors. The salary is \$1200 a month, plus 3% of the sales price of every car or truck Sid sells (this is called a *commission*).

a. The total of the sales prices of all the vehicles Sid sold during the first month on the job was \$72000. What was Sid's income (salary plus commission)?
b. In order to make \$6000 in a single month, how much selling must Sid do?
c. Write a linear equation that expresses Sid's monthly income y in terms of the value x of the vehicles Sid sold.
d. Graph this equation. What are the meanings of its y-intercept and slope?

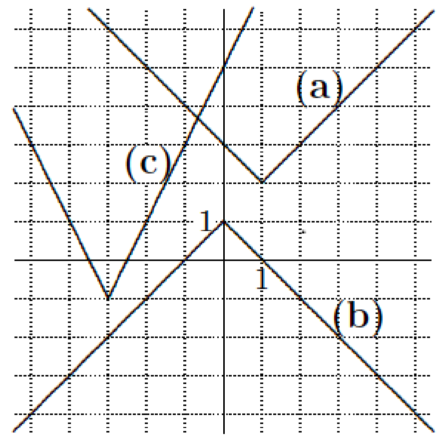
291. Find the value of x that fits the equation $1.24x - (3 - 0.06x) = 4(0.7x + 6)$.

292. At a local Candy Shop, Jess bought 5.5 pounds of candy — a mixture of candy priced at \$4 per pound and candy priced at \$3.50 per pound. Given that the bill came to \$20.75, figure out how many pounds of each type of candy Jess bought.

293. Explain how to evaluate 4^3 by hand. The superscript 3 is called an *exponent*, and 4^3 is *a power of 4*. Write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ as a power of 4. Write the product $4^3 \cdot 4^5$ as a power of 4.

294. Does every system of equations $px + qy = r$ and $mx + ny = k$ have a simultaneous solution (x, y)? Explain.

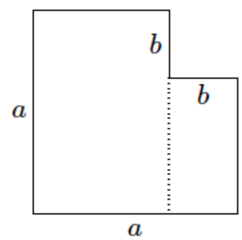
295. Write a plausible equation for each of the three graphs shown in the diagram at right.



296. Use trial-and-error to factor the following expressions

(a) x^2+2x-8 (b) x^2-x-6 (c) $2x^2+7x+6$

297. By rearranging the two parts of the diagram shown at right, show that a^2-b^2 is equivalent to $(a+b)(a-b)$.



298. The base of a rectangular tank is three feet by two feet, and the tank is three feet tall. The water in the tank is currently nine inches deep.

a. How much water is in the tank?
b. The water level will rise when a one-foot metal cube (denser than water) is placed on the bottom of the tank. By how much?
c. The water level will rise some more when a second one-foot metal cube is placed on the bottom of the tank, next to the first one. By how much?

299. Expand the following products by multiplying the two binomials:

(a) $(x-4)(x+4)$ **(b)** $(x+7)(x-7)$ **(c)** $(3x-2)(3x+2)$

300. Use the pattern to predict the factors of x^2-64 and $4x^2-25$. Explain why this pattern is called *the difference of two squares*.

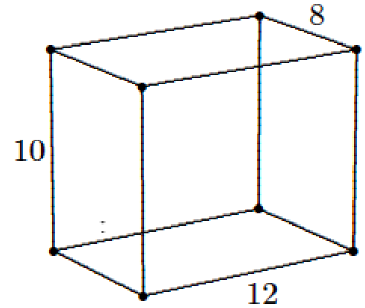
301. Wes walks from home to a friend's house to borrow a bicycle, and then rides the bicycle home along the same route. By walking at 4 mph and riding at 8 mph, Wes takes 45 minutes for the whole trip. Find the distance that Wes walked.

302. Find the intersection of $y=-2|x|+5$ and $y=|x-2|-1$. A sketch might prove helpful, but you need to justify your answer algebraically.

303. Given that s varies directly with t, and that $s = 4.56$ meters when $t = 3$ seconds, find s when t is 4.2 seconds.

304. What values of x satisfy the inequality $|x| > 12$? Graph this set on a number line, and describe it in words. Answer the same question for $|x - 2| > 12$.

305. The figure at the right shows a rectangular box whose dimensions are 8 cm by 10 cm by 12 cm.



a. Find the volume of the solid.
b. What is the combined area of the six faces?
c. If you were to outline the twelve edges of this box with decorative cord, how much would you need?

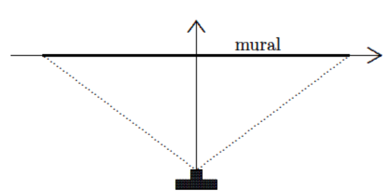
306. The area of a rectangle can be found by multiplying its dimensions. For example, if the dimensions are 2 by 7, the area is 14. Going in the other direction, one can say the factors of the area give possible dimensions for the rectangle. So, finding factors of 14 give possible dimensions of a rectangle with area 14. Similarly, to find the dimensions of a rectangle with area $x^2 + 2x - 3$ one can ask for factors of $x^2 + 2x - 3$. In this case, $x^2 + 2x - 3 = (x-1)(x+3)$ so possible dimensions are $(x-1)$ and $(x+3)$. Find possible dimensions of rectangles with area:

(a) $x^2 + 5x + 6$ **(b)** $2x^2 + 5x + 2$

307. The population of Manhattan is about 1.63 million persons.. The population of the United States is about 300 million persons. What percent of the US population lives in Manhattan? During the day, the commuter population raises that number to 3.9 million. What percent of the US population works or lives in Manhattan?

308. A rectangle is four times as long as it is wide. If its length were diminished by 6 meters and its width were increased by 6 meters, it would be a square. What are the rectangle's dimensions?

309. Using the coordinate-axis system shown in the diagram at right, the viewing area of a camera aimed at a mural placed on the x-axis is bordered by $y = \frac{7}{8}|x| - 42$. The dimensions are in feet. How far is the camera from the x-axis, and how wide a mural can be photographed?



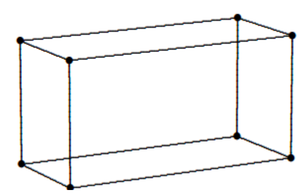
310. Factor: **(a)** $x^2 + 9x + 20$ **(b)** $3x^2 + x - 4$ **(c)** $x^2 - 81$

311. A rectangle has a width x and length $2x$ - 1. Its area is 21. Write an algebraic equation to solve for the dimensions of the rectangle.

312. After a weekend of rock-climbing in the White Mountains, Dylan is climbing down a 400-foot cliff. It takes 20 minutes to descend the first 60 feet. Assuming that Dylan makes progress at a steady rate, write an equation that expresses Dylan's height h above level ground in terms of t, the number of minutes of descending from the top. Use your equation to find how much time it will take Dylan to reach level ground.

313. Start with the equations $2x - y = 3$ and $3x + 4y = 1$. Create a third equation by adding *any* multiple of the first equation to *any* multiple of the second equation. When you compare equations with your classmates, you will probably not agree. What is certain to be true about the graphs of *all* these third equations, however?

314. The diagram at the right shows the wire framework for a rectangular box. The length of this box is 8 cm greater than the width and the height is half the length. A total of 108 cm of wire was used to make this framework.



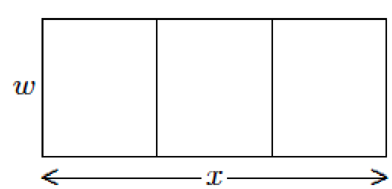
a. What are the dimensions of the box?
b. The faces of the box will be panes of glass. What is the total area of glass needed for the six panes?
c. What is the volume of the box?

315. Write a formula that expresses the distance between p and 17. Describe all the possible values for p if this distance is to be greater than 29.

316. Use the zero-product property to solve:

(a) $x^2 + 2x - 8 = 0$ **(b)** $x^2 - x - 20 = 0$ **(c)** $81 - x^2 = 0$

317. A farmer has 90 meters of fencing material with which to construct three rectangular pens side-by-side as shown at right. If w were 10 meters, what would the length x be? Find a general formula that expresses x in terms of w.



318. Find how many pairs (x, y) satisfy the equation $x + y = 25$, assuming:
   a. that there is no restriction on the values of x and y
   b. both x and y must be positive integers
   c. the values of x and y must be equal.

319. The table at the right shows the value of a car as it depreciates over time. Does this data satisfy a linear relationship? Explain.

year	*value*
1992	24 000
1993	20 400
1994	16 800
1995	13 200

320. Write an inequality that describes all the points that are more than 3 units from 5.

321. If x varies directly with y, and if $x = 5$ when $y = 27$, find x when $y = 30$.

322. The owner's manual for my computer printer states that it will print a page in 12 seconds. Re-express this speed in pages per minute, and in minutes per page.

323. A *monomial* is a constant (number) or a product of a constant and variables. If some variable factors occur more than once, it is customary to use positive integer exponents to consolidate them. Thus 12, $3ax^2$, and x^5y are monomials, but $3xy^4 + 3x^4y$ is not. Rewrite each of these monomials:

a. $x \cdot x^2 \cdot x^3 \cdot x$ b. $(2x)^7$ c. $(2w)^4(5w^2)$ d $3a^4\left(\frac{1}{2}b\right)^3ab^6$

324. Write and graph an equation that states that the perimeter of an $l \times w$ rectangle is 768 cm; that the width of an $l \times w$ rectangle is half its length.

325. (*Continuation*) Explain how the two graphs show that there is a unique rectangle whose perimeter is 768 cm, and whose length is twice its width. Find the dimensions of this rectangle.

326. You are given this system of equations

$$\begin{cases} 5x + 2y = 8 \\ x - 3y = 22 \end{cases}$$

These equations are given in a very interesting form. It is not obvious which method you should use.

a. First, find a way to use substitution to solve the system.
b. Then find a way to use linear combination to solve the system
c. Then graph the equations on Desmos and see if your answers agree.

327. My car averages 29 miles per gallon of gasoline, but I know — after many years of fueling it — that the actual miles per gallon can vary by as much as 3 either way. Write an absolute-value inequality that describes the range of possible mpg figures for my car.

328. Alex's purple superball rebounds 75% of the height from which it falls. Alex drops the ball from a height of x feet and lets it bounce. When it hits the ground for the second time, it has travelled a total of 18.75 feet. Find x. Hint: draw a picture.

329. Factor:

(a) $2x^2 + x - 21$ **(b)** $4x^2 - 15x - 4$ **(c)** $4x^2 - 81$ **(d)** $0.04x^2 - 81$

330. Solve each of the systems of equations below

(a) $\begin{cases} 3x + 4y = 1 \\ 4x + 8y = 12 \end{cases}$ (b) $\begin{cases} 2x + 3y = -1 \\ 6x - 5y = -7 \end{cases}$

331. The points (6, 4), (2, 4), and (1, 2) are on the graph of $y = a|x - h| + k$ Use an accurate graph and your knowledge of absolute-value graphs to find values for a, h, and k.

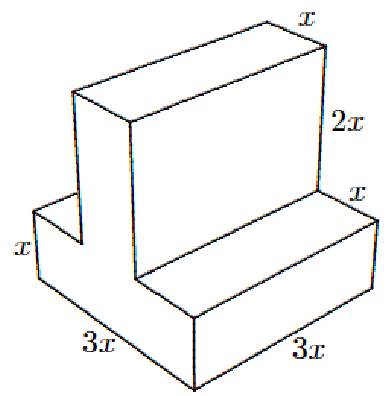
332. The difference between the length and width of a rectangle is 7 cm. Its perimeter is 50 cm. Find the length and width of this rectangle

333. Randy has 25% more money than Sandy, and 20% more money than Mandy, who has $1800. How much money does Sandy have?

334. The average of two numbers is 41. If one of the numbers is 27, what is the other number? If the average of two numbers is $x + y$, and one of the numbers is x, what is the other number?

335. The diagram at the right represents a solid of uniform cross- section (all slices perpendicular to the base are the same shape). All the lines of the figure meet at right angles. The dimensions are marked in the drawing in terms of x. Write simple formulas in terms of x for each of the following:

   a. the volume of the solid;
   b. the surface area you would have to cover in order to paint this solid;
   c. the length of decorative cord you need if you wanted to outline all of the edges of the solid.



336. A restaurant has 23 tables. Some of the tables seat 4 persons and the rest seat 2 persons. In all, 76 persons can be seated at once. How many tables of each kind are there?

337. Solve each of the following systems of equations:

(a) $\begin{cases} 3r + 5s = 6 \\ 9r = 13s + 4 \end{cases}$ (b) $\begin{cases} 3a = 1 + \frac{1}{3}b \\ 5a + b = 11 \end{cases}$

338. Use the distributive property to write each of the following in *factored form*:

(a) $ab^2 + ac^2$ **(b)** $3x^2 - 6x$ **(c)** $wx + wy + wx + w$

339. Most of Conservative Casey's money is invested in a savings account that pays 1% interest a year, but some is invested in a risky stock fund that pays 7% a year. Casey's total initial investment in the two accounts was \$10000. At the end of the first year, Casey received a total of \$250 in interest from the two accounts. Find the amount initially invested in each.

340. Find the value of p that makes the linear graph $y = p - 3x$ pass through the point where the lines $4x - y = 6$ and $2x - 5y = 12$ intersect.

341. Faced with the problem of multiplying 5^6 times 5^3, Brook is having trouble deciding which of these four answers is correct: 5^{18}, 5^9, 25^{18}, or 25^9. Your help is needed. Once you have answered Brook's question, experiment with other examples of this type until you are able to formulate the *common-base principle for multiplication* of expressions $b^m \cdot b^n$.

342. For the final in-class test in math this term, I am thinking of giving a 100-question true-false test! Right answers will count one point, wrong answers will deduct half a point, and questions left unanswered will have no effect. One way to get a 94 using this scoring system is to answer 96 correctly and 4 incorrectly (and leave 0 blank). Find another way of obtaining a score of 94.

343. (*Continuation*) Let *r* equal the number of right answers and *w* equal the number of wrong answers. Write an equation relating *r* and *w* that states that the test grade is 94.

a. Write an inequality that states that the grade is *at least* 94, and graph it.
b. Graph the inequalities $0 \le r$, $0 \le w$ and $r + w \le 100$, and explain why they are relevant here.
c. Shade the region that solves all four inequalities. How many lattice points does this region contain?
d. Why is this a lattice-point problem? What is the maximum number of wrong answers one could get and still obtain a grade at least as good as 94?

344. A large family went to a restaurant for a buffet dinner. The price of the dinner was \$12 for adults and \$8 for children. If the total bill for a group of 13 persons came to \$136, how many children were in the group?

345. Write each of the following in factored form:

(a) $2x^2 + 3x^3 + 4x^4$ **(b)** $5xp + 5x$ **(c)** $2\pi r^2 + 2\pi rh$

346. Find values for *a* and *b* that make *ax* + *by* = 14 parallel to 12 = 3*y* - 4*x*. Is there more than one answer? If so, how are the different values for *a* and *b* related?

347. Exponents are routinely encountered in scientific work, where they help investigators deal with large numbers:

a. The human population of Earth is roughly 7000000000, which is usually expressed in *scientific notation* as 7×10^9. The average number of hairs on a human head is 5×10^5. Use scientific notation to estimate the total number of human head hairs on Earth.

b. Light moves very fast — approximately 3×10^8 meters every second. At that rate, how many meters does light travel in one year, which is about 3×10^7 seconds long? This so-called *light year* is used in astronomy as a yardstick for measuring even greater distances.

348. Sage has a walking speed of 300 feet per minute. On the way to gate 14C at the airport, Sage has the option of using a moving sidewalk. By simply standing on the sidewalk, it would take 4 minutes to get to the gate that is 800 feet away.

a. How much time will it take Sage to walk the distance to the gate without using the moving sidewalk?
b. How much time will it take Sage to get to the gate by walking on the moving sidewalk?

b. After traveling 200 feet (by standing on the sidewalk), Sage notices a Moonbucks, and turns around on the moving sidewalk. How long will it take Sage to get back to the beginning of the moving sidewalk, walking in the opposite direction? Assume the sidewalk is empty of other travelers.

349. A car went a distance of 90 km at a steady speed and returned along the same route at half that speed. The time needed for the whole round trip was four hours and a half. Find the two speeds.

350. Write the following sentence using mathematical symbols: “The absolute value of the sum of two numbers *a* and *b* is equal to the sum of the absolute values of each of the numbers *a* and *b*.” Is this a true statement? Explain.

351. Fill in all of the blanks with the same number in order to make a true statement:

a. $x^2+8x+16=(x+\quad)(x+\quad)=(x+\quad)^2$

352. Solve: **(a)** $x^2+8x+16=0$ **(b)** $x^2+8x+15=0$ **(c)** $x^2+8x+7=0$

353. In a coordinate plane, shade the region that consists of all points that have positive x- and y-coordinates whose sum is less than 5. Write a system of three inequalities that describes this region.

354. Some of Lin's savings this past year were in a savings account that paid 3% interest for the year. The rest of the savings were in a riskier stock investment that paid 12% interest for the year. If Lin's \$8000 total investment yielded a return of \$600 in interest, how much was invested in the riskier account?

355. Suppose that h is 40% of p. What percent of h is p?

356. Pat is the CEO of Pat's Pickle-Packing Plant, but can still pack 18 jars of pickles per hour. Kim, a rising star in the industry, packs 24 jars per hour. Kim arrived at work at 9:00 am one day, to find that Pat had been packing pickles since 7:30 am. Later that day, Kim had packed exactly the same number of jars as Pat. At what time, and how many jars had each packed?

357. A laser beam is shot from the point $(0, 2.35)$ along the line whose slope is 3.1. Will it hit a very thin pin stuck in this coordinate plane at the point $(10{,}040, 31{,}126)$?

358. A polynomial is obtained by adding (or subtracting) monomials. Use the distributive property to rewrite each of the following polynomials in factored form. In each example, you will be finding a *common monomial factor*. A *binomial* is the sum of two unlike monomials, and a *trinomial* is the sum of three unlike monomials. The monomials that make up a polynomial are often called its *terms*.

(a) $x^2 - 2x$ **(b)** $6x^2 + 21x$ **(c)** $80t - 16t^2$ **(d)** $9x^4 - 3x^3 + 12x^2 - x$

359. The Woodman Firewood Company delivers natural firewood to homes in Manhattan. They charge a certain amount per cord for firewood and a fixed amount for each delivery, no matter how many cords are delivered. My bill from WFC last winter was \$155 for one cord of wood, and my neighbor's was \$215 for one and one-half cords. What is the charge for each cord of wood and what is the delivery charge?

360. For some cell phone plans a long-distance call costs \$2.40 plus \$0.23 per minute. If the call is not an integer number of minutes, the caller is charged \$0.23 for the fraction of a minute. Write an inequality that states that an x-minute call costs at most \$5.00. Solve the inequality to find the maximum number of minutes that it is possible to talk without spending more than \$5.00.

361. The point (2, 3) lies on the line $2x + ky = 19$. Find the value of k.

362. Taylor works after school in a health-food store, where one of the more challenging tasks is to add cranberry juice to apple juice to make a cranapple drink. A liter of apple juice costs $0.85 and a liter of cranberry juice costs $1.25. The mixture is to be sold for exactly the cost of the ingredients, at $1.09 per liter. How many liters of each juice should Taylor use to make 20 liters of the cranapple mixture?

363. Do the three lines $5x - y = 7$, $x + 3y = 11$, and $2x + 3y = 13$ have a common point of intersection? If so, find it. If not, explain why not.

364. Using an absolute-value inequality, describe the set of numbers whose distance from 4 is greater than 5 units. Draw a graph of this set on a number line. Finally, describe this set of numbers using inequalities without absolute value signs.

365. The simultaneous conditions $x - y < 6$, $x + y < 6$, and $x > 0$ define a region R. How many lattice points are contained in R?

366. Factor the following polynomials by grouping them by their greatest common factor:

(a) $x^3 - 5x^2 + 6x - 12$ **(b)** $2x^4 + 2x^2 - 3x - 9$ **(c)** $4x^2 + 20x - 3xy - 15y$

367. In $7^4 7^4 7^4 = \left(7^4\right)^x$ and $b^9 b^9 b^9 b^9 = \left(b^9\right)^y$, replace the variables with correct exponents. The expression $\left(p^5\right)^6$ means to write p^5 as a factor how many times? To rewrite this expression without exponents as $p \cdot p \cdot p \cdots$, how many factors would you need?

368. Graph the system of equations shown. What special relationship exists between the two lines? Confirm this by solving the equations algebraically. $\begin{cases} 3x - y = 10 \\ 6x = 20 + 2y \end{cases}$

369. The world is consuming approximately 87 million barrels of oil per day. At this rate of consumption, how long will the known world oil reserves of 1.653×10^{12} barrels last?

370. Uganda has recently discovered a large deposit of oil in the Lake Albert basin. It is estimated that this deposit holds as many as 6 billion barrels of oil. In how much time would this amount be consumed by worldwide demand?

371. Population data for Vermont is given in the table at right.

Year	*Population*
1970	448,327
1980	511,456
1990	564,964
2000	609,890
2010	625,741

a. Find the average annual growth rate of this population during the time interval from 1970 to 2010.
b. Write an equation for a line in point-slope form, using the ordered pair (1970*,* 448 327) and the slope you found in part (a).
c. Evaluate your equation for the years 1980 and 1990, and notice that these *interpolated* values do not agree with the actual table values. Find the size of each error, expressed as a percent of the actual population value.
d. Use your point-slope equation to *extrapolate* a population prediction for 2020.
e. New Hampshire has roughly the same area as Vermont, but its population reached one million several years ago. Predict when this will happen to Vermont's population.

372. Faced with the problem of calculating $(5^4)^3$, Brook is having trouble deciding which of these three answers is correct: 5^{64}, 5^{12}, or 5^7. Once you have answered Brook's question, experiment with other examples of this type until you are ready to formulate the principle that tells how to write $\left(b^m\right)^n$ as a power of *b*.

373. The diameter of an atom is so small that it would take about 10^8 of them, arranged in a line, to span one centimeter. It is thus a plausible estimate that a cubic centimeter contains about $10^8 \times 10^8 \times 10^8$ atoms which is equal to $\left(10^8\right)^3$ atoms. Write this huge number as a power of 10.

374. Blair runs a small stand at the local mall that sells sweatshirts. There are two types of shirts sold. One is 100% cotton, on which the markup is \$6 per shirt. The other is a cotton and polyester blend, on which the markup is \$4 per shirt. It costs Blair \$900 per month to rent the kiosk. Let *c* represent the number of pure cotton sweatshirts sold in one month and *b* the number of blended sweatshirts sold in the same month.

a. In terms of *c* and *b*, write an inequality that states that Blair's sales will at least meet the monthly rental expense. Sketch a graph.

b. This month, Blair could only get 20 of the pure cotton shirts from the distributor. This adds another constraint to the system. How does it affect the region you drew in (a)?

375. During a phone call about the system of equations, $\begin{cases} 5x+2y=8 \\ 8x+4y=8 \end{cases}$, Dylan told Max, "It's easy, just set them equal to each other." But Max replied, "That doesn't help — I get $-2y=3x$. What good is that?" Help these two students solve the problem.

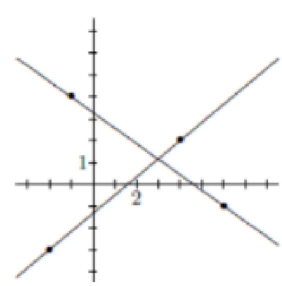
376. During 2010, it is estimated that the world consumed 5.2×10^{17} BTUs (British Thermal Units) of energy.

a. Describe this estimate of world energy use in *quadrillions* of BTUs. It is now customary to refer to one quadrillion of BTUs as simply a *quad*.

b. One barrel of oil produces about 5800000 BTUs. How many barrels of oil are needed to produce one quad?

c. The world is consuming oil at approximately 87 million barrels per day. What percentage of world energy consumption is attributable to oil?

377. The figure at right shows the graphs of two lines. Use the figure to estimate the coordinates of the point that belongs to both lines, then calculate the exact value. You will of course have to find equations for the lines, which both go through designated lattice points.



378. A math teacher is designing a test, and wants (3, -4) to be the solution to the system of equations $\{3x-5y=a, 7x+y=b\}$. What values should the teacher use for *a* and *b*?

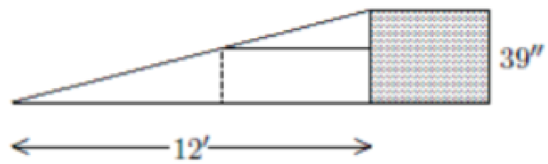
379. In each part, use the same number in each blank to make a true statement. Compare the number you put in the blanks with the original expression. What do you notice?

(a) $x^2 + 10x + 25 = (x + \_\_)(x + \_\_) = (x + \_\_)^2$

(b) $x^2 + 12x + 36 = (x + \_\_)(x + \_\_) = (x + \_\_)^2$

(c) $x^2 + 14x + 49 = (x + \_\_)(x + \_\_) = (x + \_\_)^2$

380. The figure shows a loading dock and a side view of an attached ramp, whose run is 12 feet and whose rise is 39 inches. Alex is wondering whether a long rectangular box can be stored underneath the ramp, as suggested by the dotted lines. The box is 2 feet tall and 5 feet long. Answer Alex's question.



381. Solve the system $\begin{cases} ax + ky = 1 \\ 2ax - ky = 8 \end{cases}$ for x and y in terms of a and k.

382. Sid's summer job is working at a roadside stand that specializes in homemade ice cream. The manager asks Sid to order small plain cones and extra-large sugar cones. The storage room will hold at most 12 boxes of cones. A box of small plain cones cost $30 and a box of extra-large sugar cones cost $90 dollars. A maximum of $800 is budgeted for this purchase of cones.

a. Using x for the number of boxes of plain cones and y for the number of boxes of sugar cones, translate the conditions of the problem into a system of inequalities.
b. Graph this system of inequalities and shade the feasible region for this problem. Identify the vertices of the region by specifying their coordinates.

383. Lee spent c cents to buy five pears. In terms of c and d, how many pears could Lee have bought with d dollars?

384. A catering company offers three monthly meal contracts.

a. Contract A costs a flat fee of $480 per month for 90 meals
b. Contract B costs $200 per month plus $4 per meal
c. Contract C costs a straight $8 per meal

If you expect to eat only 56 of the available meals in a month, which contract would be best for you? When might someone prefer contract A? contract B? contract C? Write a description of your reasoning with mathematical justification.

385. Graph the equation $|x + y| = 1$. Shade the region described by $|x + y| \leq 1$.

386. Sandy can saw three cords of wood in a standard workday, if the whole day is spent doing it. Sandy can split five cords of wood in a standard workday, if the whole day is spent doing it. In a standard workday, what is the largest number of cords of wood that Sandy can saw *and* split?

387. If the symbol ☆ represent some polynomial expression, factor the following in terms of ☆ .

(a) $3x☆ + 5☆$ (b) $(2x+1)☆ - 3y☆$ (c) $(4x-1)☆ - ☆(2x+2)$

388. You are buying some cans of juice and some cans of soda for your friends. The juice is \$0.60 per can while the soda is \$0.75. You have \$24, all to be spent.

a. Write an equation that represents all the different combinations of juice and soda you can buy for \$24.
b. Is it possible to buy exactly 24 cans of juice and spend the remainder on soda? Explain.
c. How many different combinations of drinks *are* possible?

389. Replace the shapes in the following two equations $\dfrac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x} = x^{\Delta}$ and $\dfrac{6^9}{6^4} = 6^{\square}$ by correct numerical exponents.

390. Jan had the same summer job for the years 1993 through 1996, earning \$250 in 1993, \$325 in 1994, \$400 in 1995, and \$475 in 1996.

a. Plot the four data points, using the horizontal axis for "year". You should be able to draw a line through the four points.
b. What is the slope of this line? What does it represent?
c. Which points on this line are meaningful in this context?
d. Guess what Jan's earnings were for 1992 and 1998, assuming the same summer job.
e. Write an inequality that states that Jan's earnings in 1998 were within 10% of the amount you guessed.

391. Now that you have dealt with systems of two-variable equations, you can apply the same principles to solve systems of three-variable equations. For example, you can (temporarily) eliminate y in the system at right: Add the first two equations, and then add the second two equations. This produces two new equations. Find x, z, and y to complete the solution. $\begin{cases} x+y+z=2 \\ x-y+z=6 \\ x+y-3z=0 \end{cases}$

392. Rewrite each of the following polynomials as a product of two factors. One of the factors should be the greatest common monomial factor.

(a) $24x^2+48x+72$ **(b)** $\pi r^2+\pi re$ **(c)** $7m-14m^2+21m^3$

393. To make a little spending money, Taylor decided to sell special souvenir programs for the Avenues Basketball Championship. The printing cost was $0.32 per program, and they were priced at $0.50 each. Taylor sold all but 50 of the programs, and made a small profit of $11. How many programs were printed?

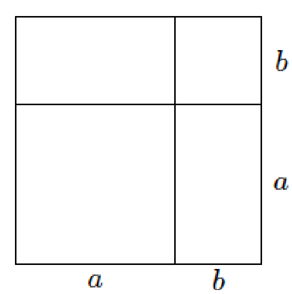
394. Factor the following polynomials as much as possible:

a. $4x^3+2x^2-2x-1$ **(b)** $24x^3-6x^2+8x-2$

395. Chet has at most 20 hours a week available to work during the summer, dividing that time between making $3 an hour babysitting and $7 an hour working for a landscaping company. Chet needs to accumulate at least $84 per week.

a. Write a system of inequalities that describes the given conditions.
b. What are the most hours Chet can work babysitting and still earn at least $84?

396. Refer to the diagram at right, which shows a large square that has been subdivided into two squares and two rectangles. Write formulas for the areas of these four pieces, using the dimensions a and b marked on the diagram. Then write an equation that states that the area of the large square is equal to the combined area of its four pieces. Do you recognize this equation?



397. Find coordinates for the point where the line $3x-2y=3001$ intersects the line $4x-3y=4001$. First solve the problem by hand, then confirm your answer using a graphing tool.

398. Solve the following for x and y in terms of a, b, and c:

$$\begin{cases} 2ax - by = 3c \\ 3ax + 2by = 2c \end{cases}$$

399. Find the equations of at least three lines that intersect each other at the point $(6,-2)$.

400. Find coordinates for the point of intersection of the lines $px + y = 1$ and $3px + 2y = 4$. You will have to express your answer in terms of p.

401. Cameron bought some 39-cent, 24-cent, and 13-cent stamps at the Post Office. The 100 stamps cost $33.40, and there were twice as many 24-cent stamps in the sale as there were 13-cent stamps. How many stamps of each denomination did Cameron buy?

402. Faced with the problem of dividing 5^{24} by 5^8 , Brook is having trouble deciding which of these four answers is correct: 5^{16}, 5^3, 1^{16} or 1^3. Your help is needed. Once you have answered Brook's question, experiment with other examples of this type until you are ready to formulate the *common-base principle for division* that tells how to divide b^m by b^n and get another power of b. Then apply this principle to the following situations:

a. Earth's human population is roughly 6×10^9, and its total land area, excluding the polar caps, is roughly 5×10^7 square miles. If the human population were distributed uniformly over all available land, approximately how many persons would be found per square mile?

b. At the speed of light, which is 3×10^8 meters per second, how many seconds does it take for the Sun's light to travel the 5×10^{11} meters to Earth?

403. Factor the following:

(a) $2x^2 - 4x$ **(b)** $x^2 + 3x$ **(c)** $2x^3 - 32x$ **(d)** $x^2 + 24x + 144$

404. Given the equation $3x + y = 6$, write a second equation that, together with the first, will create a system of equations that

a. has one solution;
b. has an infinite number of solutions;
c. has no solution;
d. has the ordered pair (4, -6) as its only solution.

405. At noon, a team bus left City College for their opponent's school. Soon thereafter, City's first-line player Brett Starr arrived at the gym. A loyal day-student parent volunteered to overtake the bus and deliver Brett. The two left at 12:15 pm. The parent drove at 54 mph, while ahead of them the yellow bus poked along at 48 mph. Did the car catch the bus before it reached the game, which is 110 miles from City college? If so, where and when?

406. Factor the following *perfect-square trinomials*:

(a) $x^2-12x+36$ **(b)** $x^2+14x+49$ **(c)** $x^2-20x+100$

407. (*Continuation*) As suggested, these should all look like either $(x-r)^2$ or $(x+r)^2$. State the important connection between the *coefficients* of the given trinomials and the values you found for *r*.

408. (*Continuation*) Complete the following to make a true statement that generalizes about perfect square trinomials and their factorization:

$$x^2+2nx+\_\_\_=(x+\_\_\_)(x+\_\_\_)=(x+\_\_\_)^2$$

409. (Continuation) In the following, choose *k* to create a perfect-square trinomial:

(a) $x^2-16x+k$ **(b)** $x^2+10x+k$ **(c)** x^2-5x+k

410. In each of the following, find the correct value for Δ:

(a) $y^4y^7=y^{\Delta}$ **(b)** $y^{12}y^{\Delta}=y^{36}$ **(c)** $y^4y^4y^4y^4=y^{\Delta}$ **(d)** $\left(y^{\Delta}\right)^3=y^{27}$

411. There are 55 ways to make $x^{■}x^{⊠}x^{⊞}=x^{12}$ a true statement. Assign positive integers to the three different squares that represent the initial exponents. Find four different ways to do it.

412. According to the US Census Bureau, the population of the USA has a net gain of 1 person every 14 seconds. How many additional persons does that amount to in one year?

413. When I ask my calculator for a decimal value of $\sqrt{1.476225}$, it displays 1.215. What is the meaning of this number? What needs to be done to check whether this square root is correct? Can $\sqrt{1.476225}$ be expressed as a ratio of whole numbers?

414. Consider the equation $y = \frac{2}{3}|x - 5| - 3$. Complete the following without using a graphing tool.

a. What are the coordinates of the vertex of this graph?
b. Find the coordinates of all axis intercepts of the graph.
c. Sketch the graph by hand.
d. Using each of these points and the vertex, compute the slope of each side of the graph. How are these slopes related?

415. The distance from Avenues to the Verrazano Narrows Bridge is 10 miles. If you walked there at 4 mph and returned jogging at 8 mph, how much time would the round trip take? What would your overall average speed be?

416. Given that three shirts cost d dollars,

a. How many dollars does one shirt cost?
b. How many dollars do k shirts cost?
c. How many shirts can be bought with q quarters?

417. What are the dimensions of a square that encloses the same area as a rectangle that is two miles long and one mile wide?

418. When I ask my calculator for a decimal value of $\sqrt{2}$, it displays 1.41421356237. What is the meaning of this number? To check whether this square root is correct, what needs to be done? Can the square root of 2 be expressed as a ratio of whole numbers — for example as $\frac{17}{12}$? Before you say “impossible”, consider the ratio $\frac{665857}{470832}$.

419. When an object falls, it gains speed. Thus the number of feet d the object has fallen is not linearly related to the number of seconds, t spent falling. In fact, for objects falling near the surface of the Earth, with negligible resistance from the air, $d = 16t^2$. How many seconds would it take for a cannonball to reach the ground if it were dropped from the top of the Eiffel Tower, which is 984 feet tall? How many seconds would it take for the cannonball to reach the ground if it were dropped from a point that is halfway to the top?

420. What happens if you try to find an intersection point for the linear graphs $3x - 2y = 10$ and $3x - 2y = -6$? What does this mean?

421. Pat and Kim are having an algebra argument. Kim is sure that x^2 is equivalent to $(x)^2$, but Pat thinks otherwise. How would you resolve this disagreement? What evidence does your calculator offer?

422. Given that Brett can wash *d* dishes in *h* hours, write expressions for

a. the number of hours it takes for Brett to wash *p* dishes;
b. the number of dishes Brett can wash in *y* hours;
c. the number of dishes Brett can wash in *m* minutes.

423. What is the value of $\frac{5^7}{5^7}$? Of $\frac{8^3}{8^3}$? Of $\frac{c^{12}}{c^{12}}$? What is the value of any number divided by itself? If you apply the common-base rule dealing with exponents and division, $\frac{5^7}{5^7}$ should equal 5 raised to what power? And $\frac{c^{12}}{c^{12}}$ should equal c raised to what power? It therefore makes sense to define c^0 to be what?

424. If $\sqrt{2}$ can be expressed as a ratio $\frac{r}{p}$ of two whole numbers, then this fraction can be put in lowest terms. Assume that this has been done.

a. Square both sides of the equation $\sqrt{2} = \frac{r}{p}$
b. Multiply both sides of the new equation by p^2. The resulting equation tells you that *r* must be an even number. Explain.
c. Because *r* is even, its square is divisible by 4. Explain.
d. It follows that p^2 is even, hence so is *p*. Explain.
e. Thus both *r* and *p* are even. Explain why this is a contradictory situation.
f. A number expressible as a ratio of whole numbers is called *rational* . All other numbers, such as $\sqrt{2}$, are called *irrational* .

425. One morning, Ryan remembered lending a friend a bicycle. After breakfast, Ryan walked over to the friend's house at 3 miles per hour, and rode the bike back home at 7 miles per hour, using the same route both ways. The round trip took 1.75 hours. What distance did Ryan walk?

426. Write the following monomials without using parentheses:

(a) $(ab)^2(ab^2)$ **(b)** $(-2xy^4)(4x^2y^3)$ **(c)** $(-w^3x^2)(-3w)$ **(d)** $(7p^2q^3r)(7pqr^4)^2$

427. The table at the right has both $y=|x|$ and $y=x^2$ evaluated at some values of x. Choose 4 four more values of x and add the y values to the table in the second two columns. Plot the points on the same system of axes. Compare your graphs with those produced by a graphing tool. In what respects are the two graphs similar? In what respects do the two graphs differ?

x	$\|x\|$	x^2
-2	2	4
-1	1	1
$-\frac{1}{2}$	0.5	0.25
0	0	0
$\frac{1}{2}$	0.5	0.25
1	1	1
2	2	4

428. A worker accidentally drops a hammer from the scaffolding of a tall building. The worker is 300 feet above the ground. As you answer the following, recall that an object falls $16t^2$ feet in t seconds (assuming negligible air resistance).

a. How far above the ground is the hammer after falling for one second? for two seconds? Write a formula that expresses the height h of the hammer after it has fallen for t seconds.
b. How many seconds does it take the hammer to reach the ground? How many seconds does it take for the hammer to fall until it is 100 feet above the ground?
c. By plotting some data points and connecting the dots, sketch a graph of h versus t. Notice that your graph is *not* a picture of the path followed by the falling hammer.

429. A box with a square base and rectangular sides is to be 2 feet and 6 inches high, and to contain 25.6 cubic feet. What is the length of one edge of the square base?

430. *Definition of a function*: Equations such as *A(x)*= $40x-x^2$ and *h(t)* $=300-16t^2$ define *quadratic functions*. The word *function* means that assigning a value to one of the variables (x or t) determines a *unique* value for the other (A or h). It is customary to say that “A is a function of x.” In this example, however, it would be incorrect to say that “x is a function of A.” Explain.

431. The graph of a quadratic function is called a *parabola*. This shape is common to all graphs of equations of the form $y = ax^2 + bx + c$, where a is nonzero. Confirm this by comparing the graph of $y = x^2$, the graph of $y = 40x - x^2$ and the graph of $y = 300 - 16x^2$.

a. How are the three graphs alike, and how are they different? Find numbers *xmin*, *xmax*, *ymin*, and *ymax*, so that the significant features of all three graphs fit in the window described by $xmin \le x \le xmax$ and $ymin \le y \le ymax$.
b. Give two examples of linear functions. Why are they called *linear* ?

432. Water pressure varies linearly with the depth of submersion. The pressure at the surface is 14.7 pounds per square inch. Given that a diver experiences approximately 58.8 pounds per square inch of pressure at a depth of 100 feet, what pressure will a submarine encounter when it is one mile below the surface of the Atlantic Ocean?

433. From the tombstone of Diophantus, a famous Greek mathematician: “God granted him to be a boy for a sixth part of his life, and, adding a twelfth part to this, He clothed his cheeks with down. He lit him the light of wedlock after a seventh part, and five years after this marriage He granted him a son. Alas! late-born wretched child — after attaining the measure of half his father’s life, chill Fate took him. After consoling his grief by his science of numbers for four more years, then did Diophantus end his life.” Calculate how old Diophantus lived to be.

434. Find the x-intercepts of each of the following quadratic graphs:

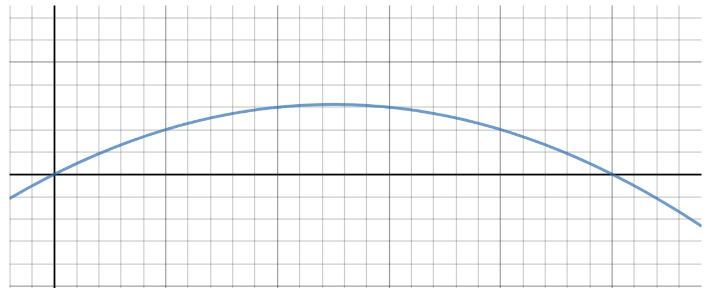
(a) $y = x^2 + 4x$ **(b)** $y = 2x^2 - 6x$ **(c)** $y = 3x^2 - 15x$ **(d)** $y = -2x^2 - 7x$

Summarize by describing how to find the x-intercepts of any quadratic graph $y = ax^2 + bx$.

435. When two rational numbers are multiplied together, their product is also a rational number. Explain. Is it necessarily true that the product of two irrational numbers is irrational? Explore this question by evaluating the following products.

(a) $\sqrt{3}\sqrt{27}$ **(b)** $\sqrt{2}\sqrt{6}\sqrt{3}$ **(c)** $\sqrt{6}\sqrt{12}$ **(d)** $\left(\sqrt{6}\right)^3$

436. *Golf math I*. Using a golf club at the 7^{th} whole, Taylor hits an excellent shot, right down the middle of the flat grass before the hole. The ball follows the parabolic path shown in the figure, described by the quadratic function $y = 0.5x - 0.002x^2$. This relates the height y of the ball above the ground to the ball's progress x down the fairway. Distances are measured in yards.



a. Use the distributive property to write this equation in factored form. Notice that $y = 0$ when $x = 0$. What is the significance of this data?
b. How far from the tee does the ball hit the ground?
c. At what distance x does the ball reach the highest point of its arc?
d. What is the highest height attained by the ball?

437. Evaluate each of the following expressions by *substituting* $s = 30$ and $t = -4$.

(a) $t^2 + 5t + s$ **(b)** $2t^2 s$ **(c)** $3t^2 - 6t - 2s$ **(d)** $s - 0.5t^2$

438. Evaluate $\sqrt{x^2 + y^2}$ using $x = 24$ and $y = 10$. Is $\sqrt{x^2 + y^2}$ equivalent to $x + y$, in this case? Does the square-root operation "distribute" over addition?

439. Evaluate $\sqrt{(x+y)^2}$ using $x = 24$ and $y = 10$. Is $(x+y)^2$ equivalent to $x + y$, in this case? Explain.

440. Evaluate $\sqrt{(x+y)^2}$ using $x = -24$ and $y = 10$. Is $\sqrt{(x+y)^2}$ equivalent to $x + y$, in this case? Explain.

441. There are several positive integers that leave a remainder of 12 when they are divided into 192. Find the smallest and the largest of those integers.

442. An Avenues student set off on a bicycle ride to the Cloisters Museum, a distance of 10 miles. After going a short while at 15 miles per hour, the bike developed a flat tire, and the trip had to be given up. The walk back to Avenues was made at a dejected 3 miles per hour. The whole episode took 48 minutes. How many miles from Avenues did the flat occur?

443. A car traveling at 60 miles per hour is covering how many feet in one second? A football field is 100 yards long. At 60 mph, how many seconds does it take to cover this distance? State your answer to the nearest tenth of a second.

444. Attempt to perform the indicated operations, and record your observations:

(a) $\sqrt{2}\sqrt{18}$ **(b)** $\sqrt{8}\sqrt{8}$ **(c)** $2\sqrt{5}\cdot 3\sqrt{20}$

Now, suggest a rule for multiplying numbers in the form $\sqrt{a}\sqrt{b}$. Extend your rule to problems in the form of $p\sqrt{a}\cdot q\sqrt{b}$.

445. (*Continuation*) Use what you have just seen to explain why $\sqrt{20}=\sqrt{4}\sqrt{5}=2\sqrt{5}$

446. Rewrite the following square roots in the same way — as the product of a whole number and a square root of an integer *that has no perfect square factors*. The resulting expression is said to be in *simplest radical form*.

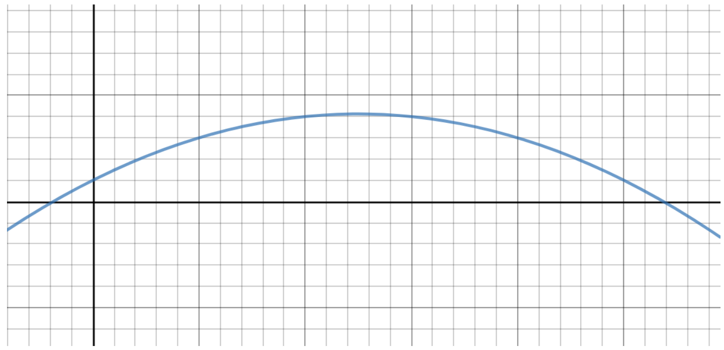
(a) $\sqrt{50}$ **(b)** $\sqrt{108}$ **(c)** $\sqrt{125}$ **(d)** $\sqrt{128}$

447. Taylor has enough money to buy either 90 granola bars or 78 pop-tarts. After returning from the store, Taylor has no money, 75 granola bars, and p pop-tarts. Assuming that Taylor has not yet eaten anything, figure out what p is.

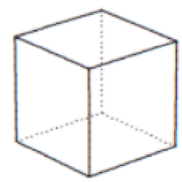
448. Avery and Sasha were comparing parabola graphs on their calculators. Avery had drawn $y=0.001x^2$ in the window $-1000\le x\le 1000$ and $0\le y\le 1000$, and Sasha had drawn $y=x^2$ in the window $-k\le x\le k$ and $0\le y\le k$. Except for scale markings on the axes, the graphs looked the same! What was the value of k?

449. At Sam’s Warehouse, a member pays \$25 a year for membership, and buys at the regular store prices. A non-member does not pay the membership fee, but does pay an additional 5% above the store prices. Under what conditions would it make sense to buy a membership?

450. *Golf math II* . Next, using the same club at the 8^{th} hole, which is on a higher piece of land 10 yards above where the 7th hole was, Taylor hits another fine shot. Explain why the quadratic function $y = 10 + 0.5x - 0.002x^2$ describes this parabolic trajectory, shown in the figure to the right. Why should you expect this tee shot to go more than 250 yards? Estimate the length of this shot, then using a graphing tool, find a more accurate value. How does this trajectory relate to the trajectory for the drive on the previous hole?



451. The total area of the six faces of a cube is 1000 sq cm. What is the length of one edge of the cube? Round your answer to three decimal places.



452. On a recent drive from New Hampshire to New York City, Taylor maintained an average speed of 50 mph for the first four hours, but could only average 30 mph for the final hour, because of road construction.

a. What was Taylor's average speed for the whole trip?
b. What would the average have been if Taylor had traveled h hours at 30 mph and $4h$ hours at 50 mph?
c. What is the average speed for a trip that consists of m miles at 30 mph followed by $4m$ miles at 50 mph?

453. Solve each of the following equations. Answers should either be exact, or else accurate to three decimal places.

(a) $x^2 = 11$ **(b)** $5s^2 - 101 = 144$ **(c)** $x^2 = 0$ **(d)** $30 = 0.4m^2 + 12$

454. You have seen that the graph of any quadratic function is a parabola that is symmetrical with respect to a line called the *axis of symmetry*, and that each such parabola also has a lowest or highest point called the *vertex*. Sketch a graph for each of the following quadratic functions. Identify the coordinates of each vertex and write an equation for each axis of symmetry.

(a) $y = 3x^2 + 6$ **(b)** $y = x^2 + 6x$ **(c)** $y = 64 - 4x^2$ **(d)** $y = x^2 - 2x - 8$

455. For the point (4, 24) to be on the graph of $y = ax^2$, what should the value of a be?

456. When asked to solve the equation $(x-3)^2 = 11$, Jess said, "That's easy — just take the square root of both sides." Perhaps Jess also remembered that 11 has two square roots, one positive and the other negative. What are the two values for x, in exact form? (In this situation, "exact" means no decimals.)

457. (*Continuation*) When asked to solve the equation $x^2 - 6x = 2$, Deniz said, "Hmm . . . not so easy, but I think that adding something to both sides of the equation is the thing to do." This is indeed a good idea, but what number should Deniz add to both sides? How is this equation related to the previous one?

458. Consider the family of quadratic functions, $ax^2 + bx + c = 0$, where a, b and c have a common difference (think about what that means…hmmmm). Graph as many of those as you need to in order to see any patterns in their graphs. What do you notice or wonder? Try to prove any conjecture you come up with.

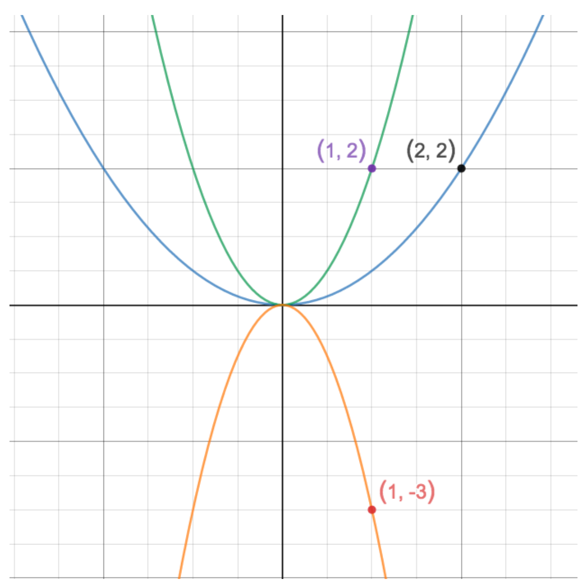
459. Use the distributive property to factor each of the following:

(a) $x^2 + x^3 + x^4$ **(b)** $\pi r^2 + 2\pi rh$ **(c)** $25x - 75x^2$ **(d)** $px + qx^2$

460. Solving a quadratic equation by rewriting the left side as a perfect-square trinomial is called solving by *completing the square*. Use this method to solve each of the following equations. Leave your answers in exact form.

(a) $x^2 - 8x = 3$ **(b)** $x^2 + 10x = 11$

(c) $x^2 - 5x - 2 = 0$ **(d)** $x^2 + 1.2x = 0.28$



461. Find a quadratic equation for each of the graphs pictured at the right. Each curve has a designated point on it, and the y-intercepts are all at integer values. Also notice that the y-axis is the axis of symmetry for all.

462. The speed of sound in air is 1100 feet per second. The speed of sound in steel is 16,500 feet per second. Robin, one ear pressed against the railroad track, hears a sound through the rail six seconds before hearing the same sound through the air. To the nearest foot, how far away is the source of that sound?

463. The point (4, 7) is on the graph of $y = x^2 + c$. What is the value of c?

464. In your notebook, use one set of coordinate axes to graph the three curves $y = x^2 - x$, $y = x^2 + 2x$ and $y = x^2 - 4x$. Make three observations about graphs of the form $y = x^2 + bx$, where b is a nonzero number.

465. Using only positive numbers, add the first two odd numbers, the first three odd numbers, and the first four odd numbers. Do your answers show a pattern? What is the sum of the first n odd numbers?

n	*odd #*	*sum*
1	1	XX
2	3	4
3	5	
4		

466. (Continuation) Copy the table at the right into your notebook and fill in the missing entries.

a. What function in terms of *x* and *y* do these points represent?

b. In the third column, compute the differences between the consecutive y coordinates.

c. Is there a pattern to the column of differences?

d. Do the values in this column describe a linear function? Explain.

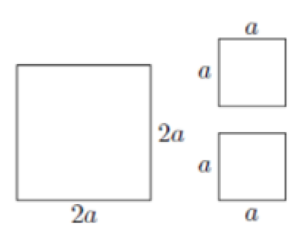
e. As a check, create a fourth column that tables the differences of the differences.

f. How does this column help you with your thinking?

x	*y*
0	0
1	1
2	4
3	9
4	
5	

467. (Continuation) Carry out the same calculations, but replace $y = x^2$ by a quadratic function of your own choosing. Is the new table of differences linear?

468. Write $(2a)^2$ without parentheses. Is $(2a)^2$ larger than, smaller than, or the same as $2a^2$? Make reference to the diagram at right in writing your answer. Draw a similar diagram to illustrate the non-equivalence of $(3a)^2$ and $3a^2$.

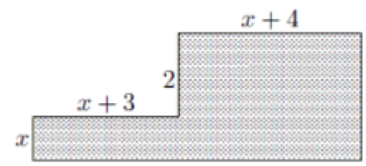


469. Solve each of the following quadratic equations by hand:

(a) $(x+4)^2 = 23$ **(b)** $7x^2 - 22x = 0$ **(c)** $x^2 + 4x = 21$ **(d)** $1415 - 16x^2 = 0$

470. A hose used by the fire department shoots water out in a parabolic arc. Let x be the horizontal distance from the hose's nozzle, and y be the corresponding height of the stream of water, both in feet. The quadratic function is $y = -0.016x^2 + 0.5x + 4.5$.

a. What is the significance of the 4.5 that appears in the equation?
b. Graph this function. Find the stream's greatest height.
c. What is the horizontal distance from the nozzle to where the stream hits the ground?
d. Will the stream go over a 6-foot high fence that is located 28 feet from the nozzle? Explain your reasoning.



471. In the diagram, the dimensions of a piece of carpeting have been marked in terms of x. All lines meet at right angles. Express the area and the perimeter of the carpeting in terms of x.

472. Evaluate the expression $397(2.598) + 845(2.598) - 242(2.598)$ mentally. Be prepared to describe your method.

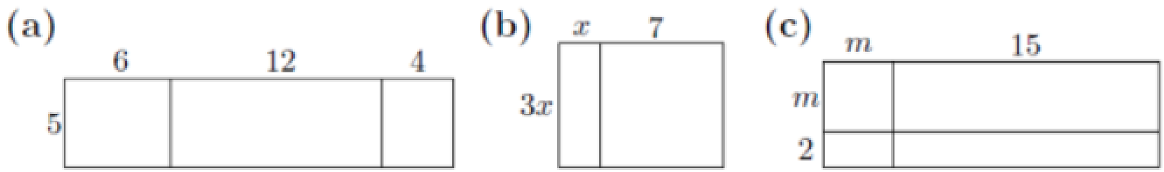
473. Kirby is four miles from the train station, from which a train is due to leave in 56 minutes. Kirby is walking along at 3 mph, and could run at 12 mph if it were necessary. If Kirby wants to be on that train, it *will* be necessary to do some running! How many miles of running?

474. The work at right shows the step-by-step process used by a student to solve $x^2 + 6x - 5 = 0$ by the method of *Completing the Square.* Explain why the steps in this process are reversible. Apply this understanding to find a *quadratic equation,* $ax^2 + bx + c = 0$, whose solutions are $x = 7 + \sqrt{6}$ and $x = 7 - \sqrt{6}$

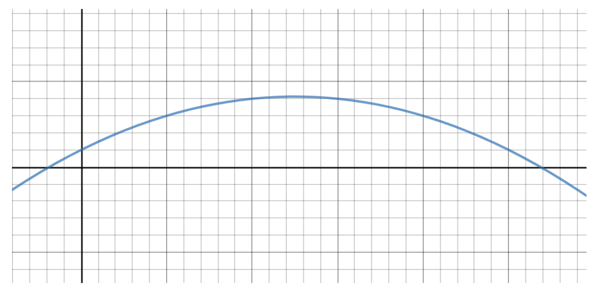
$$x^2 + 6x - 5 = 0$$
$$x^2 + 6x + 9 = 5 + 9$$
$$(x+3)^2 = 14$$
$$x + 3 = \pm\sqrt{14}$$
$$x = -3 \pm \sqrt{14}$$

475. Val hikes up a mountain trail at 2 mph. Because Val hikes at 4 mph downhill, the trip down the mountain takes 30 minutes less time than the trip up, even though the downward trail is three miles longer. How many miles did Val hike in all?

476. Express the areas of the following large rectangles in two ways. First, find the area of each small rectangle and add the expressions. Second, multiply the total length by the total width.



477. *Golf Math III.* From the graph of the path of the golf ball we can estimate the landing location of the ball. However, to find the length of the shot by hand, you must set y equal to 0 and solve for x.



a. Explain why, and show how to arrive at

$x^2 - 250x = 5000$.

b. The next step in the solution process is to add 125^2 to both sides of this equation. Why was this number chosen?

c. Complete the solution by showing that the length of the shot is $125 + \sqrt{20625}$. How does this number, which is about 268.6, compare with your previous calculation?

d. Comment on the presence of the number 125 in the answer. What is its significance?

478. If n represents a perfect square, what formula represents the next perfect square?

479. The height h (in feet) above the ground of a baseball depends upon the time t (in seconds) it has been in flight. Cameron takes a swing with the bat but sadly doesn't hit it very hard and it goes high in the air but comes down closer than he had hoped. The path of the ball could be described approximately by the equation $h = 80t - 16t^2$. Make a large, clear diagram and answer the following questions.

a. How long is the ball in the air?

b. The ball reaches its maximum height after how many seconds of flight?

c. What is the maximum height?

d. It takes approximately 0.92 seconds for the ball to reach a height of 60 feet. On its way back down, the ball is again 60 feet above the ground; what is the value of t when this happens?

480. Apply the zero-product property to solve the following equations:

(a) $(x-2)(x+3)=0$ **(b)** $x(2x+5)=0$ **(c)** $5(x-1)(x+4)(2x-3)=0$

481. Solve the following equations for x by hand.

(a) $x^2 - 5x = 0$ **(b)** $3x^2 + 6x = 0$ **(c)** $ax^2 + bx = 0$

482. During the swimming of a 50-yard sprint in a 25-yard pool, the racers swim away from the starting line and then return to it. Suppose that Alex, who always swims at a steady rate, takes 24 seconds to complete the race. Let y stand for the distance from Alex to the starting line when the race is t seconds old. Make a graph of y versus t, and write an equation for this graph.

483. In the shot-put competition at a track meet, the trajectory of Blair's best put is given by the function $h(t) = -0.0186x^2 + x + 5$, where x is the horizontal distance the shot travels, and h is the corresponding height of the shot above the ground, both measured in feet. Graph the function and find how far the shot went. What was the greatest height obtained? In the given context, what is the meaning of the "5" in the equation?

484. Sketch the graphs of $y = x^2 - 2x$, $y = -2x^2 - 14x$, and $y = 3x^2 + 18x$. Write an equation for the symmetry axis of each parabola. Devise a quick way to write an equation for the symmetry axis of any parabola $y = ax^2 + bx$. Test your method on the three given examples.

485. Simplify $\left|3 - \sqrt{5}\right| + 4$ by hand, writing an equivalent expression without absolute value signs. Do the same for $\left|3 - \sqrt{5}\right| - 4$. Does a calculator give you the same result?

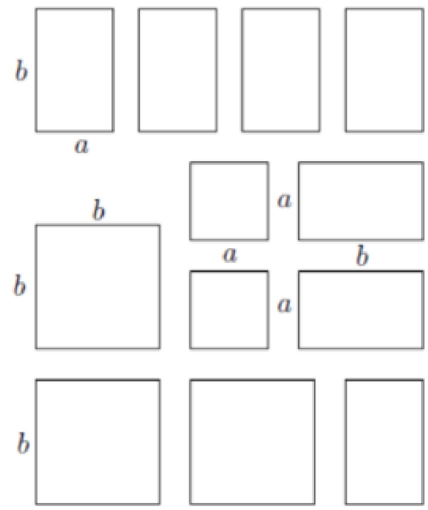
486. Multiply: **(a)** $(3x)(7x)$ **(b)** $(3x)(7 + x)$ **(c)** $(3 + x)(7 + x)$

487. Factor each expression completely.

(a) $4x^4 - 16x^2$ **(b)** $4x^2 - 18x + 8$ **(c)** $4x^2y - 19xy + 12y$

488. Given $P = (1, 4)$, $Q = (4, 5)$, and $R = (10, 7)$, decide whether or not PQR is a straight line, and give your reasons.

489. All the dimensions of the twelve rectangles in the figure are either a or b. Write an expression for the sum of the areas of the twelve pieces. This should help you to show how these twelve pieces can be fit together to form one large rectangle.



490. Sketch the graphs of $y = x^2$, $y = (x-2)^2$, $y = (x+3)^2$ and , $y = (x+5)^2$ on the same set of coordinate axes. Make a general statement as to how the graph of , $y = (x-h)^2$ is related to the graph of $y = x^2$.

491. The hands of a clock point in the same direction at noon, and also at midnight. How many times between noon and midnight does this happen?

492. (*Continuation*) When does this first happen? When is the last time this happens?

493. The axis of symmetry of a parabola is the line $x = 4$.

a. Suppose that one x-intercept is 10; what is the other one?

b. Suppose the point (12, 4) is on the graph; what other point also must be on the graph?

494. Given the equation $s = \pi r + \pi re$, solve the formula for: **(a)** e **(b)** r

495. Solve $x^2 - 2px - 8p^2 = 0$ for x in terms of p by Completing the Square.

496. (*Continuation*) Show that $x^2 - 2px - 8p^2$ can be written in factored form.

497. Find the equation of the axis of symmetry for the graph of $y = 2x^2 - 6x$. Sketch the graph of this equation in your notebook, including the axis of symmetry. What are the coordinates of the vertex of the graph?

498. (*Continuation*) Sketch the graph of $y = 2x^2 - 6x - 3$ along with its axis of symmetry. Find the coordinates of the vertex of this parabola. How do these coordinates compare with the vertex of $y = 2x^2 - 6x$? Find an equation for the graph of a quadratic curve that has the same axis of symmetry as $y = 2x^2 - 6x$ but whose vertex is at (1.5, -2.5).

499. The table at right displays some values for a quadratic function, $y = ax^2 + bx + c$.

x	0	1	2	3	4
y	0	2	6	12	20

a. Explain how to use the table to show that $c = 0$.

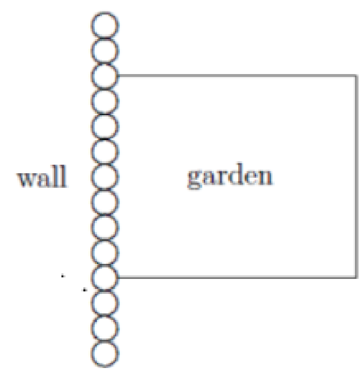
b. A point is on a curve only if the coordinates of the point satisfy the equation of the curve. Substitute the tabled coordinates (1, 2) into the given equation to obtain a linear equation in which a and b are the unknowns. Apply the same reasoning to the point (2, 6).

c. Find values for a and b by solving these two linear equations.

d. Use your values for a and b to write the original quadratic equation. Check your result by substituting the other two tabled points (3, 12) and (4, 20) into the equation.

500. Graph $y = (x - 4)^2$ and $y = 9$ using a graphing tool. What are the coordinates of the point(s) of intersection? Now solve the equation $(x - 4)^2 = 9$ by hand. Describe the connection between the points of intersection on the graph and the solution(s) to the equation.

501. Gerry Anium is designing another rectangular garden. It will sit next to a long, straight rock wall, thus leaving only three sides to be fenced. This time, Gerry has bought 150 feet of fencing in one-foot sections. Subdivision into shorter pieces is not possible. The garden is to be rectangular and the fencing (all of which must be used) will go along three of the sides as indicated in the picture.



a. If each of the two sides attached to the wall were 40 ft long, what would the length of the third side be?

b. Is it possible for the longest side of the rectangular garden to be 85 feet long? Explain.

c. Let x be the length of one of the sides attached to the wall. Find the lengths of the other two sides, in terms of x. Is the variable x *continuous* or *discrete*?

d. Express the area of the garden as a function of x, and graph this function. For what values of x does this graph have meaning?

e. Graph the line $y = 2752$. Find the coordinates of the points of intersection with this line and your graph. Explain what the coordinates mean with relation to the garden.

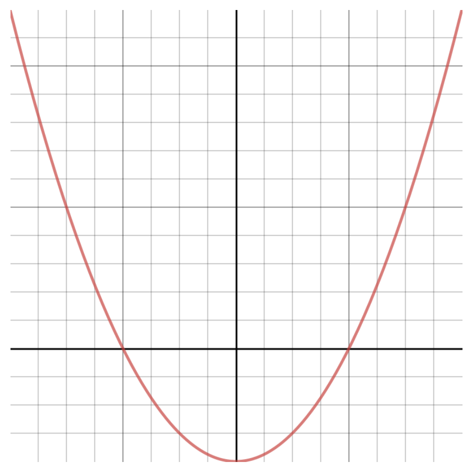
f. Gerry would like to enclose the largest possible area with this fencing. What dimensions for the garden accomplish this? What is the largest possible area?

502. Lee finds the identity $(a+b)^2 = a^2 + 2ab + b^2$ useful for doing mental arithmetic. For example, just ask Lee for the value of 75^2, and you will get the answer 5625 almost immediately — with no calculator assistance. The trick is to use algebra by letting $10k + 5$ represent a typical integer that ends with 5. Show that the square of this number is represented by $100k(k + 1) + 25$. This should enable you to explain how Lee is able to calculate $75^2 = 5625$ so quickly. Try the trick yourself: Evaluate 35^2, 95^2, and 205^2 without using a calculator, paper, or pencil.

503. Solve $x^2 + bx + c = 0$ by the method of Completing the Square. Apply your answer to the example $x^2 + 5x + 6 = 0$ by setting $b = 5$ and $c = 6$.

504. A rectangular box has length 4, width x, and height x.
   a. Express its total surface area in terms of x.
   b. If the total surface area is 66, find the width of the box.

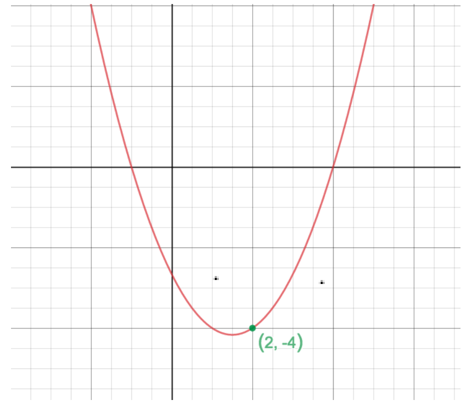
505. The graph of $y = x^2 - 400$ is shown at right. Notice that no coordinates appear in the diagram. There are tick marks on the axes, however, which enable you, without using your graphing device, to figure out the actual window that was used for this graph. Find the high and low values for both the x-axis and the y-axis. After you get your answer, check it on your device. To arrive at your answer, did you actually need to have tick marks on both axes?



506. Sketch the graph of $y = x^2 + 3$ and $y = |x| + 3$ on the same axis in your notebook. List three ways that the two graphs are alike and three ways in which they differ. Be sure your graph is large enough to clearly show these differences. On another axis, sketch the graph of $y = 2(x - 3)^2$ and $y = 2|x - 3|$. Also be prepared to explain how these two graphs compare.

507. Eighteen-carat gold contains 18 parts by weight of gold and 6 parts of other metals. (Twenty-four-carat gold is pure.) Fourteen-carat gold contains 14 parts of gold and 10 parts of other metals. How many ounces of fourteen-carat gold need to be mixed with 12 ounces of eighteen-carat gold to make seventeen-carat gold?

508. When asked to find the equation of the parabola at the right, Ryan looked at the x-intercepts and knew that the answer had to look like $y = a(x+1)(x-4)$ for some coefficient a. Justify Ryan's reasoning and then finish the solution by finding the correct value of a.



509. (Continuation) Find an equation for the parabola, in *factored form*, $y = a(x-p)(x-q)$, whose symmetry axis is parallel to the y-axis whose x-intercepts are -2 and 3, and whose y-intercept is 4. Why is factored form sometimes referred to as *intercept form*?

510. There are many quadratic functions whose graphs intersect the x-axis at (0, 0) and (6, 0). Sketch graphs for a few of them, including the one that goes through (3, 9). Other than their axis of symmetry, what do all these graphs have in common? How do the graphs differ?

511. In solving an equation such as $3x^2 - 11x = 4$ by Completing the Square, it is customary to first divide each term by 3 so that the coefficient of x^2 is 1. This transforms the equation into $x^2 - \frac{11}{3}x = \frac{4}{3}$. Now continue to solve, by the Completing the Square method, remember to take half of $\frac{11}{3}$, square it and add it to *both* sides of the equation. Finish the solution.

512. *Completing the square*. Confirm that the equation $ax^2 + bx + c = 0$ can be converted into the form $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$. Describe the steps. To achieve the goal suggested by the title, what should now be added to both sides of this equation?

513. (Continuation) The left side of the equation $x^2 + \frac{b}{a}x + \frac{b^2}{4a} = \frac{b^2}{4a} + \frac{c}{a}$ can be factored as a perfect square trinomial. Show how. The right side of the equation can be combined over a common denominator. Show how. Finish the solution of the general quadratic equation by taking the square root of both sides of your most recent equation. The answer is called *the quadratic formula*. State the quadratic formula and the two answers to the general quadratic $ax^2 + bx + c = 0$.

514. Apply your formula: Solve $x^2 + 2x - 3 = 0$ by letting $a = 1$, $b = 2$, and $c = -3$. How could you check if the answers you calculated with the quadratic formula are correct?

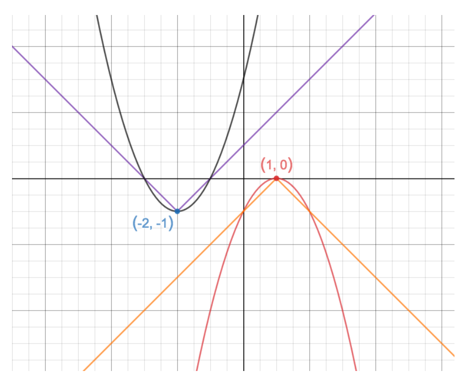
515. As long as the coefficients a and b are nonzero, the parabolic graph $y = ax^2 + bx + c$ and $y = ax^2 + bx$ has two x-intercepts. What are they? Use them to find the axis of symmetry for this parabola. Explain why the axis of symmetry for $y = 2x^2 - 5x - 12$ is the same as the axis of symmetry for $y = 2x^2 - 5x$. In general, what is the symmetry axis for $y = ax^2 + bx + c$? Does your description make sense for $y = 2x^2 - 5x + 7$, even though the curve has no x-intercepts?

516. (*Continuation)*. If you know the axis of symmetry for a quadratic function, how do you find the coordinates of the vertex? Try your method on each of the following, by first finding the symmetry axis, then the coordinates of the vertex.

(a) $y = x^2 + 2x - 3$ **(b)** $y = 3x^2 + 4x + 5$

517. Graph the equations $y = (x - 5)^2$, $y = (x - 5)^2 - 4$, and $y = (x - 5)^2 + 2$. Write the coordinates of the vertex for each curve. Describe how to transform the first parabola to obtain the other two. A fourth parabola is created by shifting the first parabola so that its vertex is (5, -7). Write an equation for the fourth parabola.

518. Find an equation for each of the functions graphed at right. Each one is either an absolute-value function or a quadratic function.



519. Simplify $\left|-\sqrt{17}+4\right| + 7$ by hand, while writing an equivalent expression without absolute-value signs. Do the same for $\left|-\sqrt{17}-4\right| - 5$ | Does your calculator agree?

520. The driver of a red sports car, moving at r feet per second, sees a pedestrian step out into the road. Let d be the number of feet that the car travels, from the moment when the driver *sees* the danger until the car has been brought to a complete stop. The equation $d = 0.75r + 0.03r^2$ models the typical panic-stop relation between stopping distance and speed. It is based on data gathered in actual physical simulations. Use it for the following:

a. Moving the foot from the accelerator pedal to the brake pedal takes a typical driver three fourths of a second. What does the term $0.75r$ represent in the stopping-distance equation? The term $0.03r^2$ comes from physics; what must it represent?
b. How much distance is needed to bring a car from 30 miles per hour (which is equivalent to 44 feet per second) to a complete stop?
c. How much distance is needed to bring a car from 60 miles per hour to a complete stop?
d. Is it true that doubling the speed of the car doubles the distance needed to stop it?

521. (*Continuation*) At the scene of a crash, an officer observed that a car had hit a wall 150 feet after the brakes were applied. The driver insisted that the speed limit of 45 mph had not been broken. What do you think of this evidence?

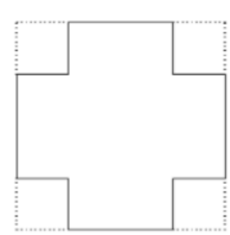
522. Perform the indicated operations and combine like terms where possible:

(a) $(x+6)(x-7)$ **(b)** $(x-5)^2$ **(c)** $(x+9)(x-9)$

523. Sketch the graphs of $y = (x - 4)^2$ and $y = (4 - x)^2$. What do you notice about the graphs? Explain why this is true.

524. Jess bought a can of paint, whose label stated that the contents of the can were sufficient to cover 150 square feet. The surface that Jess wants to paint is a square, each edge of which is n inches long. Given that n is a whole number, how large can it be?

525. The diagram at the right suggests an easy way of making a box with no top. Start with a square piece of cardboard, cut squares of equal sides from the four corners, and then fold up the sides. Here is the problem: To produce a box that is 8 cm deep and whose capacity is exactly one liter (1000 cc). How large a square must you start with (to the nearest mm)?



526. Solve the following quadratic equations:

(a) $x^2 + 6x + 5 = 0$ **(b)** $x^2 - 7x + 12 = 0$ **(c)** $3x^2 + 14x + 8 = 0$ **(d)** $2x^2 + 5x - 3 = 0$

527. The three functions $y = 2(x - 4) - 1$, $y = 2|x - 4| - 1$, and $y = 2(x - 4)^2 - 1$ look somewhat similar. Predict what the graph of each will look like, and then sketch them by hand by plotting a few key points. In each case, think about how the form of the equation can help provide information.

528. Make a sketch of the parabola $y = (x - 50)^2 - 100$, by finding the x-intercepts, the y-intercept, and the coordinates of the vertex. Label all four points with their coordinates on your graph.

529. When taking an algebra quiz, Casey was asked to factor the trinomial $x^2 + 3x + 4$. Casey responded that this particular trinomial was not factorable. Decide whether Casey was correct, and justify your response.

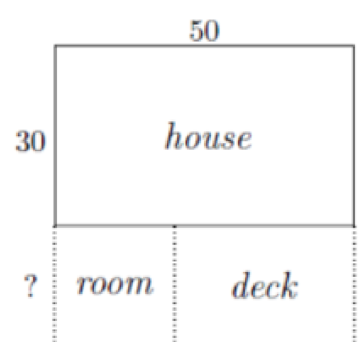
530. The graph of a quadratic function intersects the x-axis at 0 and at 8. Draw two parabolas that fit this description and find equations for them. How many examples are possible?

531. Find an equation for the parabola whose x-intercepts are 0 and 8, whose axis of symmetry is parallel to the y-axis, and whose vertex is at

(a) (4, -16) **(b)** (4, -8) **(c)** (4, -4) **(d)** (4, 16)

532. Find the value for c that forces the graph of $3x + 4y = c$ to go through (2, -3).

533. Consider the graph of $y = x^2 + 1$. Notice that it has no x-intercepts. Assume you did not know that and try to find them in the usual way by letting $y = 0$ and solving for x. What happens?



534. Pat and Kim own a rectangular house that measures 50 feet by 30 feet. They want to add on a family room that will be square, and then fill in the space adjoining the new room with a deck. A plan of the setup is shown at right. They have not decided how large a family room to build, but they do have 400 square feet of decking. If they use it all, and keep to the plan, how large will the family room be? Is there more than one solution to this problem?

535. Write in as compact form as possible:

(a) $x\dfrac{1}{x^2}$ **(b)** $\left(\dfrac{2}{x^3}\right)^4$ **(c)** $(2x+x+2x)^3$ **(d)** $\dfrac{x^6}{x^2}$

536. Write each of the following quadratic functions in factored form. Use this form to find x-intercepts for each function and use the intercepts to sketch a graph by hand. Include the coordinates for each vertex.

(a) $y=x^2-4x-5$ **(b)** $y=x^2+12x+35$ **(c)** $y=x^2-3x+2$

537. (Continuation) In the previous problem, expressing a polynomial in factored form made it relatively easy to graph the polynomial function. Here we explore the process in reverse; that is, try using the graph of a polynomial function to factor the polynomial. In particular, use a graphing tool to draw $y=x^3-3x^2-x+3$ and from that graph deduce the factored form.

538. By using square roots, express the solutions to $(x-5)^2-7=0$ exactly (no decimals).

539. Find the x-intercepts of the following graphs, without expanding the squared binomial that appears in each:

(a) $y=(x-4)^2-9$ **(b)** $y=-2(x+3)^2+8$

Check your work by sketching each parabola, incorporating the vertex and x-intercepts.

540. Consider the parabola $y=x^2+1$.

a. Sketch a graph of this parabola. Does it have any x-intercepts?
b. Suppose you want to find the x-intercepts without first considering its graph. What happens when you try to solve the equation $0=x^2+1$?

541. No real number can be squared to give a negative number. However, there are solutions to equations such as $x^2=-1$ if we consider a different set of numbers. This new set of numbers is called *complex numbers*. In this new set of numbers, $\sqrt{-1}$ is actually defined and it is called i. This means $i^2=-1$ by definition. The powers of i can be simplified to expressions without exponents (other than 1). Simplify:

(a) i^3 **(b)** i^4 **(c)** i^5

542. The *degree* of a monomial counts how many variable factors would appear if it were written without using exponents. For example, the degree of $6ab$ is 2, and the degree of $25x^3$ is 3, since $25x^3 = 25xxx$. The degree of a polynomial is the largest degree found among its monomial terms. Find the degree of the following polynomials:

(a) $x^2 - 6x$ **(b)** $5x^3 - 6x^2$ **(c)** $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$ **(d)** $4\pi r^2 h$

543. Find at least three integers (positive or negative) that, when put in the blank space, make the expression $x^2 + \_\_\_ x - 36$ a factorable trinomial. Are there other examples? How many?

544. (Continuation) Find at least three integers that, when put in the blank space, make the expression $x^2 + 4x - \_\_\_\_$ a factorable trinomial. Are there other examples? How many? What do all these integers have in common?

545. Jessie walks a certain distance to a friend’s house at 4 miles per hour and returns by bicycle on a route that is 8 miles longer. Jessie bikes at an average rate of 12 miles per hour. If the total time for the round trip is 90 minutes, what is the exact distance Jessie walked?

546. Plot a point near the upper right corner of a sheet of graph paper. Move your pencil 15 graph-paper units (squares) to the left and 20 units down, then plot another point. Use your ruler to measure the distance between the points. Because the squares on your graph paper are probably larger or smaller than the squares on your classmates’ graph paper, it would not be meaningful to compare ruler measurements with anyone else in your class. You should therefore finish by converting your measurement to graph-paper units.

547. (*Continuation*) Square your answer (in graph-paper units), and compare the result with the calculation $15^2 + 20^2$.

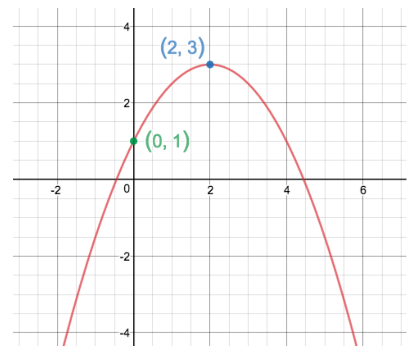
548. (*Continuation*) Repeat the entire process, starting with a point near the upper left corner, and use the instructions “20 squares to the right and 21 squares down.” You should find that the numbers in this problem again fit the equation $a^2 + b^2 = c^2$. These are instances of the *Pythagorean Theorem*, which is a statement about right-angled triangles. Write a clear statement of this useful result. You will need to refer to the longest side of a right triangle, which is called the *hypotenuse*.

549. A cylindrical container is filled to a depth of d cm by pouring in V cc of liquid. Draw a plausible graph of d versus V. Recall that *d versus V* means that V is on the horizontal axis.

550. The product of two polynomials is also a polynomial. Explain. When a polynomial of degree 3 is multiplied by a polynomial of degree 2, what is the degree of the result?

551. Consider the parabola $y = x^2 + 2x + 3$.
   a. Sketch its graph. Does it have any x-intercepts?
   b. What happens when you try to solve the equation $0 = x^2 + 2x + 3$ to find the x-intercepts?

552. When asked to find the equation of the parabola pictured at right, Ryan reasoned that the correct answer had to look like $y = a(x-2)^2 + 3$, for some value of a. Justify Ryan's reasoning, then finish the problem by finding the correct value of a.



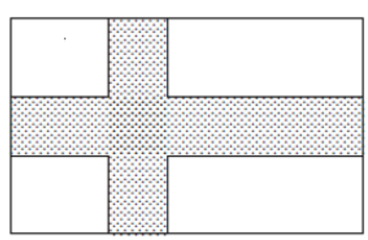
553. Find an equation for the parabola whose symmetry axis is parallel to the y-axis, whose vertex is (-1, 4) and whose graph contains the point (1, 3).

554. Starting at school, you and a friend ride your bikes in different directions — you ride 4 blocks north and your friend rides 3 blocks west. At the end of this adventure, how far apart are you and your friend?

555. From the library, you ride your bike east at a rate of 10 mph for half an hour while your friend rides south at a rate of 15 mph for 20 minutes. How far apart are you? How is this problem similar to the preceding problem? How do the problems differ?

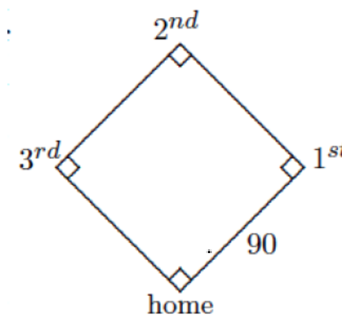
556. Imagine a circle of rope, which has twelve evenly spaced knots tied in it. Suppose that this rope has been pulled into a taut, triangular shape, with stakes anchoring the rope at knots numbered 1, 4, and 8. Make a conjecture about the angles of the triangle.

557. The diagram at right shows the flag of Finland, which consists of a blue cross, whose width is a uniform 9 inches, against a solid white background. The flag measures 2 feet 9 inches by 4 feet 6 inches. The blue cross occupies what fractional part of the whole flag?



558. Combine over a common denominator: **(a)** $\dfrac{1}{x-3}+\dfrac{2}{x}$ **(b)** $\dfrac{1}{x-3}+\dfrac{2}{x+3}$

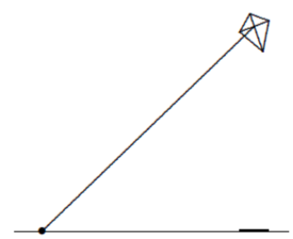
559. In baseball, the infield is a square that is 90 feet on a side, with bases located at three of the corners, and home plate at the fourth. If the catcher at home plate can throw a baseball at 70 mph, how many seconds does it take for the thrown ball to travel from home plate to second base?



560. Graph the equation $y = (x - 5)^2 - 7$ by hand by plotting its vertex and x-intercepts (just estimate their positions between two consecutive integers). Then use a graphing tool to draw the parabola. Repeat the process on $y = -2(x+6)^2 + 10$. How close was your hand-plotting to the actual graph?

561. At most how many solutions can a quadratic equation have? Give an example of a quadratic equation that has two solutions. Give an example of a quadratic equation that has only *one* solution. Give an example of a quadratic equation that has *no* solutions.

562. While flying a kite at the beach, you notice that you are 100 yards from the kite's shadow, which is directly beneath the kite. You also know that you have let out 150 yards of string. How high is the kite?



563. The sides of Fran's square are 5 cm longer than the sides of Tate's square. Fran's square has 225 sq cm more area. What is the area of Tate's square?

564. Graph the three points (-2, 1), (3, 1), and (0, 7). There is a quadratic function whose graph passes through these three points. Sketch the graph. Find its equation in two ways:

   a. First, begin with the equation of the parabola in standard form and use the three points to find the *a*, *b* and *c*.
   b. Second, begin with the equation of the parabola in the vertex form and use the three points to determine *a*, *h*, and *k*. (One of these values is almost given to you).
   c. Your two equations do not look alike, but they should be. With some algebra, show that they are the same equation.

565. Solve each of the following by the method of completing the square:

(a) $3x^2 - 6x = 1$ **(b)** $2x^2 + 8x - 17 = 0$

566. Use a calculator to evaluate the following: **(a)** $\dfrac{\sqrt{50}}{\sqrt{2}}$ **(b)** $\dfrac{\sqrt{28}}{\sqrt{7}}$ **(c)** $\dfrac{\sqrt{294}}{\sqrt{6}}$

567. Explain why your results make it reasonable to write $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$. Check that this rule also works for: **(a)** $\dfrac{\sqrt{48}}{\sqrt{6}}$ **(b)** $\dfrac{\sqrt{84}}{\sqrt{12}}$ **(c)** $\dfrac{\sqrt{180}}{\sqrt{15}}$

568. *Eliminating an Irrational Number in the Denominator:* How are the decimal approximations for $\dfrac{6}{\sqrt{6}}$ and $\sqrt{6}$ related? Was this predictable? Why? Verify that the decimal approximations for $\dfrac{1}{\sqrt{8}}$ and $\dfrac{\sqrt{2}}{4}$ are equal. Was the answer predictable? What is the effect of multiplying $\dfrac{1}{\sqrt{8}}$ and $\dfrac{\sqrt{2}}{\sqrt{2}}$? To show equivalence of expressions, you might have to transform one radical expression to make it look like another by simply multiplying by the number 1 in the form of a fraction. This is sometimes called *Rationalizing the Denominator.*

569. (Continuation) By hand, decide whether the first expression is equivalent to the second:

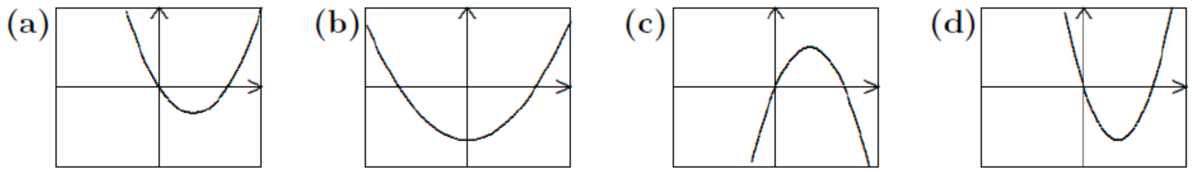
(a) $\sqrt{75}$ and $5\sqrt{3}$ **(b)** $\dfrac{\sqrt{800}}{2}$ and $10\sqrt{2}$ **(c)** $\dfrac{2}{\sqrt{8}}$ and $\dfrac{\sqrt{2}}{2}$ **(d)** $\sqrt{\dfrac{1000}{6}}$ and $\dfrac{10\sqrt{15}}{3}$

570. In each of the following, supply the missing factor:

(a) $2x^2 + 5x - 12 = (2x - 3)(\qquad)$ **(b)** $3x^2 - 2x - 1 = (3x + 1)(\qquad)$

(c) $4y^2 - 8y + 3 = (2y - 1)(\qquad)$ **(d)** $6t^2 - 7t - 3 = (3t + 1)(\qquad)$

571. Which of the following screens could be displaying the graph of $y = x^2 - 2x$? To support your answer, explain what portion of the x-axis and y-axis are shown?



572. Graph the equation $y = -2x^2 + 5x + 33$. For what values of x
(a) is $y = 0$? **(b)** is $y = 21$? **(c)** is $y \geq 0$?

573. The expression $4x + 3x$ can be combined into one term, but $4x + 3y$ cannot. Explain. Similar rules apply to radical terms.

a. Can $4\sqrt{5} + 2\sqrt{5}$ be combined into one term? If so, what is it? If not, why not?
b. Can $\sqrt{2} + \sqrt{2}$ be combined into one term? If so, what is it? If not, why not?
c. Can $3\sqrt{5} - 2\sqrt{3}$ be combined into one term? If so, what is it? If not, why not?
d. At first glance it may seem that $\sqrt{2} + \sqrt{8}$ cannot be combined into one term. Take a close look at $\sqrt{8}$ and show that $\sqrt{2} + \sqrt{8}$ can in fact be combined into one term.

574. Because $\sqrt{8}$ can be rewritten as $2\sqrt{2}$ the expression $\sqrt{8} + 5\sqrt{2}$ can be combined into a single term $7\sqrt{2}$. By hand, combine each of the following into one term.
(a) $\sqrt{12} + \sqrt{27}$ **(b)** $\sqrt{63} - \sqrt{28}$ **(c)** $\sqrt{6} + \sqrt{54} + \sqrt{150}$ **(d)** $2\sqrt{20} - 3\sqrt{45}$

REFERENCE SECTION

absolute value:	The absolute value of x is denoted $\|x\|$ and is the distance between x and zero on a number line. The absolute value of a quantity is never negative.
additive inverse:	See *opposite*.
average speed:	The average speed during a time interval is $\frac{\textit{total change in distance}}{\textit{total change in time}}$
average a list of numbers:	Add all values in list and divide by how many numbers in the list (also known as Mean)
axis of symmetry:	A line that separates a figure into two parts that are equivalent by reflection across the line. Every *parabola* has an axis of symmetry.
balance diagram:	A diagram displaying a scale that is in equilibrium.
binomial:	The sum of two unlike monomials, *e.g.* $x + 2$ or $3x^3y - 7z^5$.
British Thermal Unit:	A BTU is a unit of energy, approximately the amount needed to raise the temperature of a gallon of water by 1 degree Celsius.
cc:	Abbreviation for cubic centimeters. See *conversions*.
Celsius:	A scale for recording temperatures. It is defined by the stipulation that water freezes at 0 degrees and boils at 100 degrees.
coefficient:	See *monomial* .
collinear:	Three (or more) points that all lie on a single line are collinear.
combine over a common denominator:	To create a single fraction that is equal to a given sum of fractions.

commission:	This is an extra payment to a salesperson for making a sale above their salary.
Common base principle for division:	This states that to divide exponents (or powers) with the same base, subtract the exponents. Example: $2^9 \div 2^4 = 2^5$
common denominator:	Given a set of fractions, a common denominator is divisible by every one of the given denominators.
common monomial factor:	A *monomial* that divides every term of a *polynomial* .
completing the square:	Adding a quantity to a trinomial so that the new trinomial can be factored as a perfect square.
conjecture:	An unproven statement that seems likely to be true.
consecutive integers:	Two integers are consecutive if their difference is 1.
continuous:	A variable whose values fill an *interval* . Continuous variables represent quantities that are divisible, such as time and distance. See also *discrete*.
conversion factors:	values that relate different units by representing how many of each are equivalent. Some examples are: 1 mile = 5280 feet; 1 foot = 12 inches; 1 inch = 2.54 centimeters; one liter is 1000 milliliters; a milliliter is the same as a cubic centimeter.
coordinate:	A number that locates a point on a number line or describes the position of a point in the plane with respect to two number lines (axes).
degree:	For a monomial, this counts how many variable factors would appear if the monomial were written without using exponents. The degree of a polynomial is the largest degree found among its monomial terms. Example: $x^2 + 5x^3 - x + 20$ has degree 3.

dependent variable: When the value of one variable determines a unique value of another variable, the second variable is sometimes said to *depend* on the first variable. See also *function.*

difference of two squares: a squared (multiplied by itself) number subtracted from another squared number. Every difference of squares may be factored according to the identity $a^2 - b^2 = (a + b)(a - b)$

direct variation: Two quantities *vary directly* if one quantity is a constant multiple of the other. Equivalently, the ratio of the two quantities is constant. The graph of two quantities that vary directly is a straight line passing through the origin.

discrete: A variable that can only take on integer values and only a finite number of them. See also *continuous*.

distributive property: Short form of "multiplication distributes over addition," a special property of arithmetic. In algebraic code: $a(b+c)$ and $ab+ac$ are equivalent, as are $(b+c)a$ and $ba + ca$, for any three numbers a, b, and c. Multiplication also distributes over subtraction, of course.

endpoint convention: If an interval includes an endpoint (as in $6 \leq x$ or $y \geq 4$), this point is denoted graphically by filling in a circle. If an interval excludes an endpoint (as in $6 < x$ or $y < -4$), this point is denoted by drawing an empty circle.

equation: A statement that two expressions are equivalent. For example, $3x + 5 = 2x - 4$, and $(x + 3)^2 = x^2 + 6x + 9$ are equations. Note: there are big differences between formulas, expressions and equations.

An equation that has all numbers as solutions is called an *identity* (the second equation is an example of this).

evaluate: Find the numerical value of an expression by *substituting* numerical values for the *variables*. For example, to evaluate $2t + 3r$ when $t = 7$ and $r = -4$, substitute the values 7 and -4 for t and r, respectively.

exponent: An integer that indicates the number of equal factors in a product. For example, the exponent is 3 in the expression w^3, which means $w \cdot w \cdot w$.

exponents, rules of: These apply when there is a *common base*: $a^m \cdot a^n = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$. An exponential expression is raised to a power: $(a^m)^n = a^{mn}$. Notice the special case of the common-base rules: $a^0 = 1$.

extrapolate: To enlarge a table of values by going outside the given range of data usually using a linear model created by the given data.

factor: *Noun*: a number or expression that divides another number or expression without remainder. For example, 4 is a factor of 12, $2x$ is a factor of $4x^2 + 6xy$.

Verb: to rewrite a number or an expression as a product of its factors. For example, 12 can be factored as $2 \cdot 2 \cdot 3$, and $4x^2 + 6xy$ can be factored as $2x(2x + 3y)$.

factored form: Written as a product of factors. For example, $(x-3)(2x + 5) = 0$ is written in factored form. If an equation is in factored form it is particularly easy to find the solutions, which are $x = 3$ and $x = -\frac{5}{2}$ (See *Zero Product Property*)

factored form of a quadratic function: For variables x and y, and real numbers a, p and q, with $a \neq 0$, the equation $y = a(x - p)(x - q)$ is commonly called factored form or *intercept form of a quadratic function*.

Fahrenheit: A scale for recording temperatures. It is defined by the stipulation that water freezes at 32 degrees and boils at 212 degrees.

feasible region: A region of the plane defined by a set of inequalities. The coordinates of any point in the feasible region satisfy all the defining inequalities.

function: A function is a rule that describes how the value of one quantity (the dependent variable) is determined by the value of another quantity (the independent variable). A function can be defined by a formula, a graph, or a table.

function notation: A function is described by a lowercase letter, usually around *f* in the alphabet, and the independent variable, usually x, explicitly set equal to the rule for that function. For example, $f(x) = 3x+7$ or $g(x) = x^2-2x$

greatest common (integer) factor: Given a set of integers, this is the largest integer that divides all of the given integers. Also called the *greatest common divisor* .

greatest common (monomial) factor: Given a set of *monomials*, this is the largest monomial that divides all of the given monomials.

guess-and-check: A method for creating equations to solve word problems. In this approach, the *equation* emerges as the way to check a *variable* guess. Initial practice is with constant guesses, so that the checking can be done with ordinary arithmetic.

hypotenuse: In a right triangle, the side opposite the right angle. This is the longest side of a right triangle.

identity: An *equation*, containing at least one *variable*, that is true for all possible values of the variables that appear in it. For example, $x(x + y) = x^2 + xy$ is true no matter what values are assigned to x and y.

income: See *revenue*.

inequality: A statement that relates the positions of two quantities on a number line. For example, $5 < x$ or $t \leq 7$.

integer: The whole number and their opposites, that is, the set $\{. . .-3, -2, -1, 0, 1, 2, 3, . . .\}$.

intercept form of aquadratic function: see *factored form of a quadratic function*

interpolate: To enlarge a table of values by staying within the given range of data.

intersection point: See *point of intersection*.

interval: A connected piece of a number line. It might extend infinitely far in the positive direction (as in $-1 < x$), extend infinitely far in the negative direction (as in $t \leq 7$), or be confined between two endpoints (as in $2 < m \leq 7$).

irrational number: A number that cannot be expressed exactly as the ratio of two integers. Two familiar examples are π and $\sqrt{2}$. See *rational number* .

lattice point: A point both of whose coordinates are integers. The terminology derives from the rulings on a piece of graph paper, which form a lattice.

light year: Approximately 5.88 trillion miles, this is a unit of length used in astronomical calculations. As the name implies, it is the distance traveled by light during one year.

like terms: These are *monomials* that have the same variables, each with the same exponents, but possibly different numerical coefficients. Like terms can be combined into a single monomial; unlike terms cannot.

linear: A polynomial, equation, or function of the first degree. For example, $y = 2x - 3$ defines a linear function, and $2x + a = 3(x - c)$ is a linear equation.

lowest terms: A fraction is in lowest terms if the greatest common factor of the numerator and denominator is 1. 6/3 is not in lowest terms because 6 and 3 have 3 as a common factor.

model: An equation (or equations) that describe a context quantitatively.

monomial: A constant (real number) or a product of a constant and variables. In the case when the monomial is not simply a constant, the constant part is called the *coefficient*. Any exponents of variables are restricted to be non-negative integers. For example: 3, x^3, y^3x^2, and $3x^5$ are monomials. See also *binomial* , *polynomial* , and *trinomial* .

number line: A line on which two points have been designated to represent 0 and 1. This sets up a one-to-one correspondence between numbers and points on the line.

opposite: When the sum of two quantities is zero, they are called opposites (or *additive inverses*); each is the opposite of the other. On a number line, zero is exactly midway between any number and its opposite.

or: Unless you are instructed to do otherwise, interpret this word *inclusively* in mathematical situations. Thus a phrase ". . . (something is true) or (something else is true) . . . " allows for the possibility that *both* (something is true) and (something else is true).

parabola: The shape of a graph of the form $y = ax^2 + bx + c$. All parabolas have a *vertex* and an *axis of symmetry*

Perfect Square Trinomial: This is a trinomial that can be factored into a binomial multiplied by itself. (a single binomial squared)

perimeter: The total length of the sides of a figure. The perimeter of a rectangle is twice the length plus twice the width. In algebraic code, $p = 2l + 2w = 2(l + w)$.

period of a pendulum: The time needed for a pendulum to swing back and forth once.

point of intersection: A point where one line or curve meets another. The coordinates of a point of intersection must satisfy the equations of the intersecting lines.

point-slope form: The line with slope m that passes through the point (h, k) can be described in point-slope form by either $y - k = m(x - h)$ or $y = m(x - h) + k$.

polynomial: A sum of *monomials*. See also *binomial* and *trinomial*.

profit: The result of deducting total costs from total *revenues*.

proportion: An *equation* stating that two *ratios* are equal. $\frac{3}{4} = \frac{6}{8}$ is a proportion.

Pythagorean Theorem: The square of the length of the *hypotenuse* of a right triangle equals the sum of the squares of the lengths of the other two sides.

quadratic equation: a polynomial equation of degree 2

quadratic formula: The solution to the quadratic equation $ax^2 + bx + c = 0$, which can be written as $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

quadratic function: A function defined by an equation of the form $y = ax^2 + bx + c$, where y is the dependent variable. The word quadratic comes from a Latin word that means “to make square”, and it refers to the presence of a squared variable in the equation.

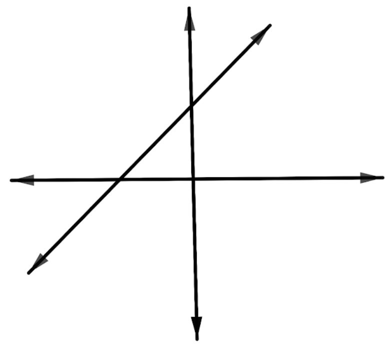
quadrillion: To an English speaker, this is $1000000000000000 = 1.0 \times 10^{15}$

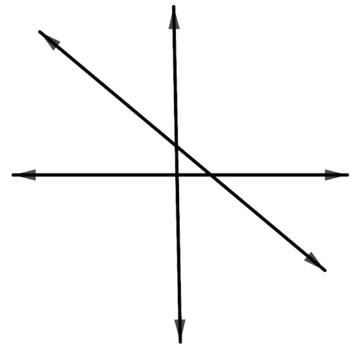
radical expression: An expression containing roots, like $2 + \sqrt{3}$

rate (of change): Rate often denotes speed, *i.e.* units of distance per unit of time. For example, 60 miles per hour, 50 feet per second, 67 furlongs per fortnight. However, the rate does not have to represent a speed, it can be any measure of change of a quantity per amount of another quantity. For example, 5 liters per student, 24 angels per pinhead, 1.3 thousand persons per year, 70 passengers per lifeboat.

ratio: The ratio of *a to b* is the expression $\frac{a}{b}$;also written $a:b$ or a/b or $a \div b$.

rational number:	A number that can be written as the ratio of two integers. See also *irrational number.*
reciprocal:	When the product of two quantities is 1, they are called reciprocals (or *multiplicative inverses*); each is the reciprocal of the other. For example, 0.2 is the reciprocal of is the reciprocal of 5. Any nonzero number has a reciprocal.
relatively prime integers:	have no common divisor that is larger than 1.
revenue:	This is money received as a result of sales; also known as *income*.
Scandinavian flags:	Flags that are all based on the *Dannebrog*.
scatter plot:	The graph of a discrete set of data points.
scientific notation:	The practice of expressing numbers in the form $a\,x10^n$, in which n is an integer, and a is a number whose magnitude usually satisfies $1 \leq \|a\| < 10$.
simplest radical form:	An expression ab is in simplest radical form if b is a positive integer that no factors that are perfect squares. For example, $18\sqrt{5}$ is in simplest radical form, but $18\sqrt{75}$ is not.
simultaneous solution:	A solution to a system of equations must satisfy *every* equation in the system.
slope:	The slope of a line is a measure of its steepness. It is computed by the ratio of the $\frac{change\ in\ y\ coordinates}{change\ in\ x\ coordinates}$. Geometrically, often it is seen as the ratio of $\frac{rise}{run}$ of the line. A line with positive slope has increasing y values for increasing x values. A line with negative slope has decreasing y values for increasing x values.





slope-intercept form: The line whose slope is m and whose y-intercept is b can be described in slope-intercept form by $y = mx + b$.

solve: To find the numerical values of the variables that make a given equation or inequality a true statement. Those values are called *solutions*.

square: To multiply a number by itself; *i.e.* b^2 is the square of b.

square root: A square root of a nonnegative number k is a number whose square is k. If k is positive, there are two such roots. The positive root is denoted $\sqrt{k}$, and sometimes called "the square root of k." The negative root is denoted $-\sqrt{k}$.

standard form: A linear equation in the form $ax + by = c$. Notice that this refers to a linear equation, which should not be confused with the standard *form of a quadratic function*.

standard form of a quadratic function: A function that has degree 2, has the graph of a parabola. When it is in the form of a trinomial, with coefficients a and b and constant c, the quadratic is said to be in standard form $y = ax^2 + bx + c$.

substitution: Replacing one algebraic expression by another of equal value.

system of equations: A set of two or more equations. The solution to a system of linear equations is the coordinates of the point where the lines meet. The solution is the values of the variables that satisfy all the equations of the system at the same time.

trial-and-error factoring: Factoring a trinomial $ax^2 + bx + c$ into the product of two binomials $(px + q)(rx + s)$ by using trial-and-error to find numbers p, q, r, and s such that $pr = a$, $qs = c$, and $ps + qr = b$.

triangular number: Any integer obtained by summing $1 + 2 + ...+ n$, for some positive integer n.

trinomial: The sum of three unlike monomials, e.g. $x^2 + 2x + 3$ or $10q + 3qr - 5r^2$

variable: A letter (such as x, y, or n) used to represent a number. A few letters (such as
m and n) tend to be associated with integers, but this is not a rule.

versus: This was once the name of a television sports network. It is also a word that frequently appears when describing graphs, as in "the graph of volume versus time." This book follows the convention of associating the first-named variable with the vertical axis, and the second-named variable with the horizontal axis. The first-named variable is *dependent* on the second-named variable.

vertex: A "corner" point on an absolute-value graph. The vertex of the graph $y = a|x - h| + k$ is the point (h, k). The vertex of the graph of a quadratic function is the point whose y-coordinate is extreme (highest or lowest). It is the point on the parabola that is also on the axis of symmetry. The vertex of the graph $y = a(x - h)^2 + k$ is (h, k).

vertex form of a quadratic function: The function of degree 2 that has a graph of a parabola In general it can be written as a trinomial but can be rewritten after completing the square in the form : $y = a(x - h)^2 + k$ is commonly called the vertex form of a quadratic function. The ordered pair (h, k) denotes the coordinates of the vertex.

whole numbers:	The numbers $\{0, 1, 2, 3, ...\}$. [65, 586, 596, 606, 632]
x-intercept:	The x-coordinate of a point where a line or curve meets the x-axis. The terminology is sometimes applied to the point itself.
y-intercept:	The y-coordinate of a point where a line or curve meets the y-axis. The terminology is sometimes applied to the point itself.
zero-product property:	If the product of a set of factors is zero, then at least one of the factors must be zero. In symbols, if $ab = 0$ then either $a = 0$ or $b = 0$.