Avenues: Math 2
2020-21

Adapted from original materials by
Phillips Exeter Academy
2016-2017

Introduction for the Student:

Members of the Avenues School and Phillips Exeter Academy Mathematics Department have created the material in this book. As you work through it, you will discover that algebra, geometry, and trigonometry have been integrated into a mathematical whole. There is no Chapter 5, nor is there a distinct section on right triangles. The curriculum is problem-based, rather than chapter-oriented.

A major goal of this course is have you practice thinking mathematically and to learn to become a more independent and creative problem solver. Problem solving techniques, new concepts and theorems will become apparent as you work through the problems, and it is your classroom community's responsibility to make these conclusions together. Your responsibility is to keep appropriate notes for your records — there are no boxes containing important theorems. There is no index as such, but the reference section at the end of the problems should help you recall the meanings of key words that are defined in the problems (where they usually appear italicized).

I. The Mathematical Thinking Process

1. Stay/Think/Say/Draw
    a. Reading each question carefully and repeatedly is essential, especially since definitions, highlighted in italics, are routinely inserted into the problem texts. Check the reference section regularly.
    b. It is important to make accurate diagrams whenever appropriate.
2. Talk/Use Resources
    a. Talk out loud, speak to friends, ask questions, email your teacher
    b. Your prior knowledge – what you know already or have forgotten that you know – is your best resource.
    c. Use your notes, the internet
3. Estimate
    a. Before you try any mathematical formulas at all, you should have some idea of what the answer should be – is really large like 3000? Or should it be something small like .05?
4. Mathematize
    a. Formulas(Pythagorean theorem, quadratic formula, equations of lines), concepts (area, linear motion, what a triangle is, the sum of the angles in a triangle) and rules of mathematics (two points determine a line, all numbers squared are positive) can be used at this point in the process.
5. Try/Refine/Revise
    a. If something does work, see why it didn't work
    b. Change the method
    c. Try something else!

II. **<u>Problem Solving as Homework</u>**: You should approach each problem as an **<u>exploration.</u>** **You are not expected to come to class every day with every problem completely solved. Your presentations in class are expected to be unfinished solutions.**

- Useful strategies to keep in mind are:
    - create an easier problem
    - guess and check
    - work backwards
    - make use of prior knowledge
    - recall a similar problem.
- It is important that you work on each problem when assigned, since the questions you may have about a problem will likely motivate class discussion the next day. In other words, doing homework to get ahead is not a good idea since class discussion will help you prepare for future problems.
- Try to justify each step you do – ask *why* not just *how*. Justification is more important than the answer on a nightly basis.
- Problem-solving requires persistence as much as it requires ingenuity. When you get stuck, or solve a problem incorrectly, back up and start over. Keep in mind that you're probably not the only one who is stuck, and that may even include your teacher.
- If you have taken the time to think about a problem, <u>you should bring to class a written record of your efforts</u>, not just a blank space in your notebook. There should be a diagram, equation, reference to similar problem, evidence of your work or questions you had on the problem. This is what will get you credit for doing your homework!!

The methods that you use to solve a problem, the corrections that you make in your approach, the means by which you test the validity of your solutions, and your ability to communicate ideas are just as important as getting the correct answer. <u>You are not to spend more than the allotted time for that night's homework on any one nightly assignment, so please manage your study hall time carefully!</u>

Most importantly, be patient with yourself – learning to problem solve independently takes time, courage and practice.

III. **<u>About technology</u>**: Many of the problems in this book require the use of technology (graphing calculators or computer software) in order to solve them. Moreover, you are encouraged to use technology to explore, and to formulate and test conjectures. Keep the following guidelines in mind:

- write before you calculate, so that you will have a clear record of what you have done
- store intermediate answers in your calculator for later use in your solution
- pay attention to the degree of accuracy requested
- be prepared to explain your method to your classmates, including bringing your laptop to class with the file on it (or emailing it to your teacher the night before) in order to project your solution to the class

<u>IV. Classroom Contribution:</u>

Learning in a PBL classroom is very different for most students for many different reasons. What is valued in the PBL classroom and what is considered successful takes time to understand, so most importantly you should come with an open mind and be ready to openly communicate. Be sure to communicate your learning needs to your teacher throughout the year. Here are some comments from past students:

About presenting homework solutions:

"The fact that we have to get up in front of the class helped in my learning"

"The accumulative mixture of problems the book had really helped me see the connections between concepts"

"I got more comfortable with taking mathematical risks"

"This curriculum has made me a better problem solver"

"It helped challenge me and taught me even if I didn't think I was learning"

"Make sure you try all of the problems – even if you can't get them."

About communication in class:

"I loved being able to discuss issues with classmates"

"It helps when the teacher summarizes what we learn"

"I liked finding more than one way to do something"

About getting support:

"Meeting with my teacher really helped"

"Asking questions is a sign of strength not weakness"

"I liked how it was focused on yourself figuring out the problem – though that was hard for me to adjust to – however it's made me much more independent math-wise"

Becoming a better independent problem solver is not an easy journey, but it does need your whole-hearted curiosity and effort. The mathematics department is here to support you through this year so please make use of the support systems that are available if you feel you need them.

1. Show by finding examples that it is hardly ever true that $\sqrt{a^2 + b^2}$ is the same as $\sqrt{a} + \sqrt{b}$.

2. Expand each of the following expressions and collect like terms:

   (a) $(x+2)^3$ **(b)** $(x+3)(x^2-3x+9)$ **(c)** $1-(x+1)^2$ **(d)** $(2x+1)^2-2(x+1)^2$

3. What is the exact value of the expression x^2 - 5 when $x = 2 + \sqrt{5}$. Be prepared to show your work.

4. Find the x-intercepts in exact form of each of the following graphs:

   (a) $y=(x-6)^2-10$ **(b)** $y=3(x-7)^2-9$

   (c) $y=120-3x^2$ **(d)** $y=4.2-0.7x^2$

5. In each of the following collect like terms if possible:

   (a) $7\sqrt{6}+3\sqrt{6}$ **(b)** $13\sqrt{3}-5\sqrt{3}$ **(c)** $\sqrt{32}-\sqrt{72}$ **(d)** $\sqrt{243}+\sqrt{48}-\sqrt{108}$

6. Given that $\sqrt{k}=\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}$, find the value of k by hand.

7. Show that $x=3+\sqrt{2}$ is a solution to the equation $x^2-6x+7=0$.

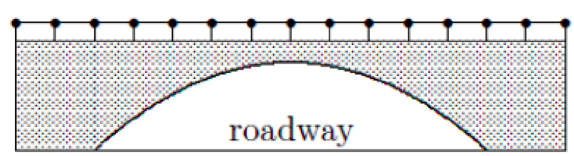
8. Expand and simplify the following products of two factors:

   (a) $(x-1)(x+1)$ **(b)** $(x-1)(x^2+x+1)$ **(c)** $(x-1)(x^3+x^2+x+1)$

9. Factor each of the following as completely as you can:

(a) p^4-4p^2 **(b)** w^3-2w^2-15w **(c)** $16y-9yz^2$ **(d)** $2x^2+20x+50$

10. The figure shows a bridge arching over the Laconic Parkway. To accommodate the road beneath, the arch is 100 feet wide at its base, 20 feet high in the center, and parabolic in shape.

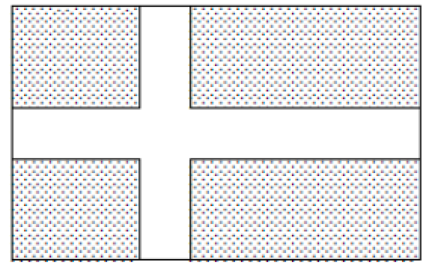


   a. The arch can be described by $y=kx(x-100)$, if the origin is placed at the left end of the arch. Find the value of the coefficient k that makes the equation fit the arch.
   b. Is it possible to move a rectangular object that is 40 feet wide and 16.5 feet high (a wide trailer, for example) through the opening? Explain.

11. Sam is a guest on the TV show *Math Jeopardy*, and has just chosen the $300 question in the category "Quadratic Equations." The answer is "The solutions are $x = 3$ and $x = 2$." What question could Sam ask that would win the $300? Is there more than one possible correct question?

12. If the points (4, 2) and (-3, -2) were the endpoints of the hypotenuse of a right triangle, how long would its legs be? Draw a diagram and then solve for the length of the hypotenuse.

13. The diagram at right shows the flag of Sweden, which consists of a gold cross of uniform width against a solid blue background. The flag measures 3 feet 4 inches by 5 feet 4 inches, and the area of the gold cross is 30% of the area of the whole flag. Use this information to find the width of the gold cross.



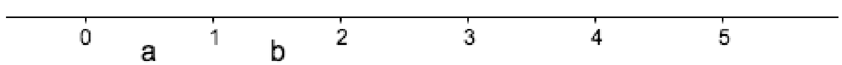
14. Calculate the following distances, and briefly explain your method:
    (a) from (2, 1) to (10, 10) **(b)** from (-2, 3) to (7, -5)
    (c) from (0, 0) to (9, 8) **(d)** from (4, -3) to (-4, 6)

15. Pat and Kim are having another algebra argument. Pat is quite sure that $\sqrt{x^2}$ is equivalent to x, but Kim thinks otherwise. How would you resolve this disagreement?

16. To get from one corner of a rectangular court to the diagonally opposite corner by walking along two sides, a distance of 160 meters must be covered. By going diagonally across the court, 40 meters are saved. Find the dimensions of the court, to the nearest cm.

17. On the number line shown below, a is a number between 0 and 1, and b is a number between 1 and 2.

    Mark possible positions on this line for $\sqrt{a}$, $\sqrt{b}$, a^2, b^2 and $\sqrt{\dfrac{a}{b}}$.



18. The Ninth Grade class is going to produce a yearbook covering their first year, compiled from photos and stories submitted by Ninth Graders. The printing company charges $460 to set up and print the first 50 copies; additional copies are $5 per book. Only books that are paid for in advance will be printed (so there will be no unsold copies), and no profit is being made.
    a. What is the cost to print 75 copies? What is the selling price of each book?
    b. Write a function that describes the cost of printing n copies, assuming that $n \geq 50$.
    c. Express the selling price of each book as a function of n, assuming that $n \geq 50$.
    d. The Ninth Graders want to sell the book for $6.25. How many books must be sold to do this?

19. What is the meaning of the number k when you graph the equation $y = mx + k$? What is the meaning of the number k when you graph the equation $x = my + k$?

20. A triangle has $K = (3, 1)$, $L = (-5, -3)$, and $M = (-8, 3)$ for its vertices. Verify that the lengths of the sides of triangle KLM fit the Pythagorean equation $a^2 + b^2 = c^2$.

21. A rectangle has an area of 36 square meters. Its length is 23 meters. In exact form, what is the perimeter of the rectangle?

22. How far is the point (5, 5) from the origin? Find two other first-quadrant lattice points that are *exactly* the same distance from the origin as (5, 5) is.

23. By hand, find the value of $x^3 - 2x^2y + xy^2$ when $x = 21$ and $y = 19$.

24. At noon one day, AJ decided to follow a straight course in a motorboat. After one hour of making no turns and traveling at a steady rate, the boat was 5 miles east and 12 miles north of its point of departure. What was AJ's position at two o'clock? How far had AJ traveled? What was AJ's speed?

25. Wes and Kelly decide to test their new walkie-talkies, which have a range of six miles. Leaving from the spot where Kelly is standing, Wes rides three miles east, then four miles north. Can Wes and Kelly communicate with each other? What if Wes rides another mile north? How far can Wes ride on this northerly course before communication breaks down?

26. We know that the axis of symmetry for a parabola in the form $y = ax^2 + bx + c$ can be found from the formula $x = -\dfrac{b}{2a}$. The equation of the axis of symmetry can help us find the y coordinate of the vertex. Make the appropriate substitution, using $x = -\dfrac{b}{2a}$, and find a formula for the y-coordinate of the vertex in terms of a, b, and c.

27. (Continuation) Find the x-intercepts of $y = a(x-h)^2 + k$ in terms of a, h, and k.

28. (Continuation) Using the fact that $x = h$ is the axis of symmetry and k is the y- coordinate of the vertex, make substitutions in your x-intercept formulas to express the x-intercepts in terms of a, b, and c, rather than h and k. Does your answer remind you of another important formula in algebra?

29. Alex is making a 4-mile trip. The first two miles were at 30 mph. At what speed must Alex cover the remaining two miles so that the average speed for the entire trip will be:
(a) 50 mph? **(b)** 55 mph? **(c)** 59.9 mph? **(d)** 60 mph?

30. Consider the equation $3x^2 - 2x + 1 = 0$.
    a. Solve it.
    b. Are the solutions real numbers?
    c. What does this tell you about the graph of $y = 3x^2 - 2x + 1$?

31. Give an example of a line that is parallel to $2x + 5y = 17$. Describe your line by means of an equation. Which form for your equation is most convenient? Now find an equation for a line that is equidistant from your line and the line $2x + 5y = 41$.

32. A rowing team training on the Hudson River, which has a current of 3 kph, wondered what their speed r would be in still water. A mathematician in the boat suggested that they row two timed kilometers — one going upstream and one going downstream. Write an expression that represents their total time rowing these two kilometers, in terms of r.

33. Most positive integers can be expressed as a sum of two or more consecutive positive integers. For example, 24 = 7 + 8 + 9, 36 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8, and 51 = 25 + 26. A positive integer that cannot be expressed as a sum of two or more consecutive positive integers is therefore *interesting*. The simplest example of an interesting number is 1.
    a. Show that no other odd number is interesting.
    b. Show that 14 is not an interesting number.
    c. Show that 82 is not an interesting number.
    d. Find three ways to show that 190 is not an interesting number.
    e. Find three ways to show that 2004 is not an interesting number.
    f. How many interesting numbers precede 2004?

34. On a single set of coordinate axes, graph several parabolas of the form $y = bx - x^2$. Mark the vertex on each curve. What do you notice about the configuration of all such vertices?

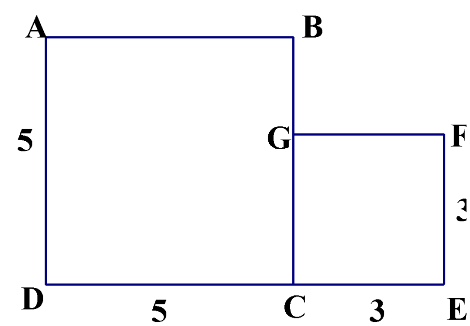
    Sketch the graphs of $y = 2\sqrt{x}$ and $y = x - 3$, and then find all points of intersection. Now solve the equation $2\sqrt{x} = x - 3$ by first squaring both sides of the equation. Do your answers agree with those obtained from the graph?

35. Consider the solutions to $ax^2 + bx + c = 0$. What must be true in order for the solutions to be real numbers?

36. You are given the two squares and the sheets of paper (see handouts). Your task is to cut the two given squares into pieces and reform those pieces into a larger square. (Hint: Cut out the entire outline of the two squares, and cut along the dotted line shown, but do not cut the dashed line between the squares).
    a. Are you sure that the new shape that you formed is in fact a square? Can you justify that to your neighbor? What is enough evidence and reasoning? Write down your argument and then compare with another set of classmates.
    b. In class you will compare your "puzzle" solution to others' methods. See if what you did to form the third square is the same.

37. Let $A = (0, 0)$, $B = (7, 1)$, $C = (12, 6)$, and $D = (5, 5)$. Plot these points and connect the dots to form the quadrilateral $ABCD$. Verify that all four sides have the same length. Such a figure is called *equilateral*.

38. If the hypotenuse of a right triangle is 12 and one of the legs is 4, what is the length of the other leg? What is the simplest form in which you can express your answer?



39. A 5×5 square and a 3×3 square can be cut into pieces that will fit together to form a third square.
    (a) Where would you place a point P, such that AP and PF would be the lines to cut to form the third square?
    (b) What is the area of the third square?

40. (Continuation) Change the sizes of the squares to $AD = 8$ and $EF = 4$, and redraw the diagram. Where should point P be marked this time? What is the side length of the third square?

41. (Continuation) Will the preceding method *always* produce pieces that form a new square? If your answer is *yes*, prepare a written explanation. If your answer is *no*, provide a counterexample – two specific squares that can *not* be converted to a single square.

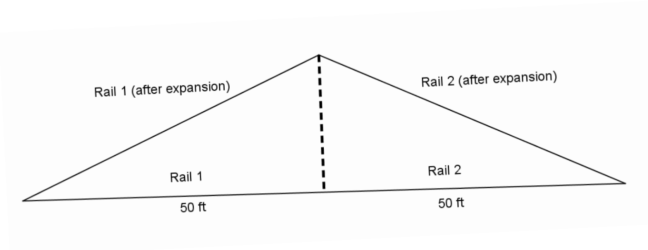
42. The main use of the Pythagorean Theorem is to find distances. Originally (6^{th} century BC), however, it was regarded as a statement about *areas*. Explain this interpretation.

43. Instead of walking along two sides of a rectangular field, Fran took a shortcut along the diagonal, thus saving distance equal to half the length of the longer side. Find the length of the long side of the field, given that the length of the short side is 156 meters.

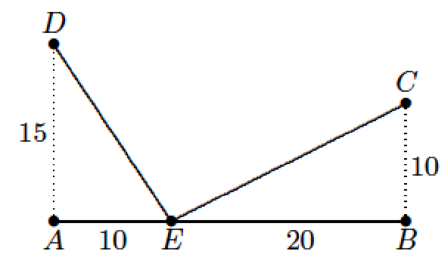
44. Two different points on the line $y = 2$ are each exactly 13 units from the point $(7, 14)$. Draw a picture of this situation, and then find the coordinates of these points.

45. Consider two adjacent squares with side lengths a and b. What is the length of the side of the third square formed by the puzzle solution method we have used previously?

46. Two iron rails, each 50 feet long, are laid end to end with no space between them. During the summer, the heat causes each rail to increase in length by 0.04%. Although this is a small increase, the lack of space at the joint makes the joint buckle upward. What distance upward will the joint be forced to rise? [Assume that each rail *remains straight*, and that the other ends of the rails are anchored.]



47. In the diagram *AEB* is a straight line and angles *A* and *B* are right. Calculate the total distance $DE + EC$ in simplest radical form.



48. Given $A = (5, -3)$ and $B = (0, 6)$. Find a point *C* that makes angle *ACB* a right angle. Find the hypotenuse of the right triangle formed.

49. Give an example of a point that is the same distance from (3, 0) as it is from (7, 0). Find lots of examples. Describe the configuration of all such points. In particular, how does this configuration relate to the two given points?

50. Verify that the hexagon formed by $A = (0, 0)$, $B = (2, 1)$, $C = (3, 3)$, $D = (2, 5)$, $E = (0, 4)$, and $F = (-1, 2)$ is equilateral. Is it also *equiangular*?

51. Draw a 20-by-20 square *ABCD*. Mark *P* on *AB* so that $AP = 8$, *Q* on *BC* so that $BQ = 5$, *R* on *CD* so that $CR = 8$, and *S* on *DA* so that $DS = 5$. Find the lengths of the sides of quadrilateral *PQRS*. Is there anything special about this quadrilateral? Explain.

52. Find two points that are not horizontal nor vertical from each other that are exactly five units apart.

53. The general notation in geometry is that points are labeled with capital letters and coordinates are defined with lowercase letters. Given the two points $A = (x_1, y_1)$ and, $B = (x_2, y_2)$ what do the subscripts on *x* and *y* represent? If triangle *ABC* is a right triangle with *C* being the right angle:

    a. Find possible coordinates for point *C* in terms of the coordinates for *A* and *B* that are given.
    b. How could you express the length of the side *BC*? *AC*? *AB*?

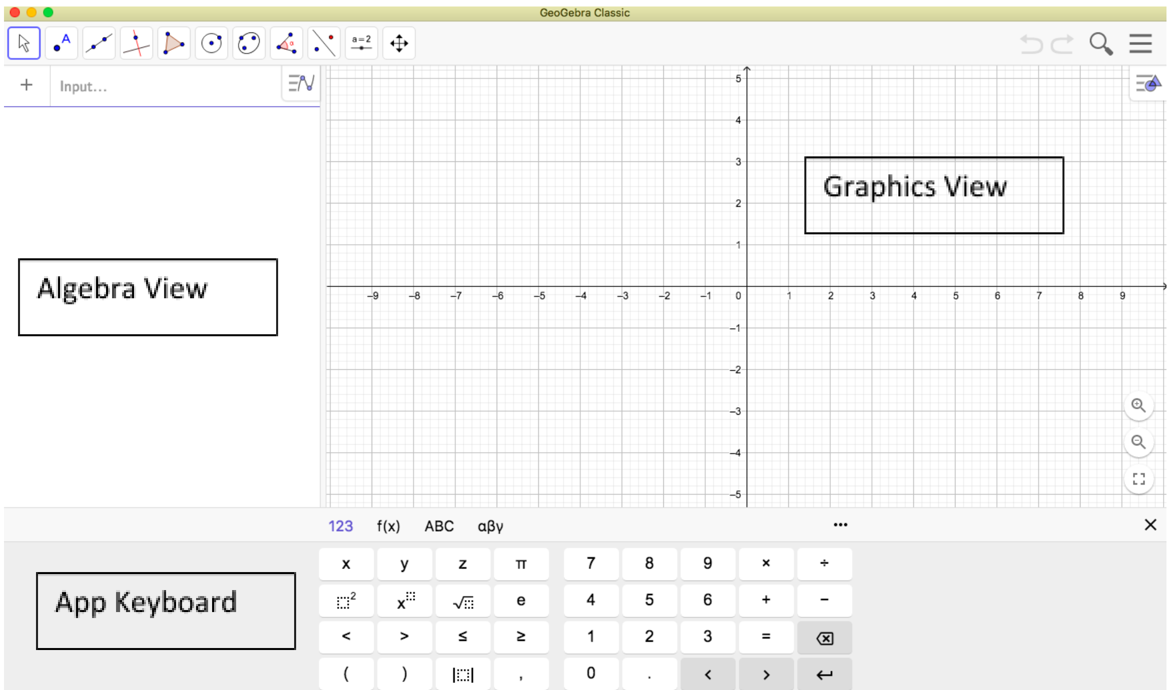
54. Verify that $P = (1,-1)$ is the same distance from $A = (5, 1)$ as it is from $B = (-1, 3)$. It is customary to say that *P* is *equidistant* from *A* and *B*. Find three more points that are equidistant from *A* and *B*. By the way, to "find" a point means to find its *coordinates*. Can points equidistant from *A* and *B* be found in every *quadrant*?

55. If you were writing a geometry book, and you had to define a mathematical figure called a *kite*, how would you word your definition?

<u>GeoGebra Activity 1 – Getting Comfortable with GeoGebra</u>

GeoGebra is an open-source dynamic geometry and algebra app. The first thing you will need to do is download GeoGebra on your computer. To do this go to the Apple App Store on your computer.

1. Search for **GeoGebra Classic 6.** Download this on your laptop. Make sure you are downloading <u>GeoGebra Classic 6</u> because there are many different GeoGebra apps.
2. When you open GeoGebra for the first time you should see three different sections in the app window.



3. You can resize any of these three windows but the Keyboard is not necessary if you are using the app on your laptop, so you can click on the *x* in the top right hand corner.
4. Try plotting some points on the Graphics View by typing some coordinates in the Algebra View. **In the "Input Bar" type, (5, 3), (2, -1) and (9, 0).** You should notice that when you type the open parenthesis GeoGebra automatically knows that you want to type an ordered pair and puts the closed parenthesis there for you. Also notice that GeoGebra automatically labels points with capital letters in alphabetical order. If you need specific labels, you right click on the point and choose "Rename."
5. **Rename the three points on your screen G, E and O.**
6. You will notice 11 boxed icons at the top left of the app screen. These are known as "Toolboxes" as there are drop down menus that have specific tools that you can use in the Graphics View to construct geometric or other objects or to do other actions. In the "line toolbox" which is the third from the left, there are many construction tools. **Use the segment tool to draw three segments to create Triangle GEO in the Graphics View.**
7. What do you see now in the Algebra View? What kind of triangle is GEO? How do you know?
8. **Go to the measurement toolbox which is the fourth from the right and choose the angle measuring tool.** There are many ways to measure angles. The easiest is probably to click on the two sides of the angle you desire the measurement of in a counterclockwise order. **Measure all three of the angles of triangle GEO.**

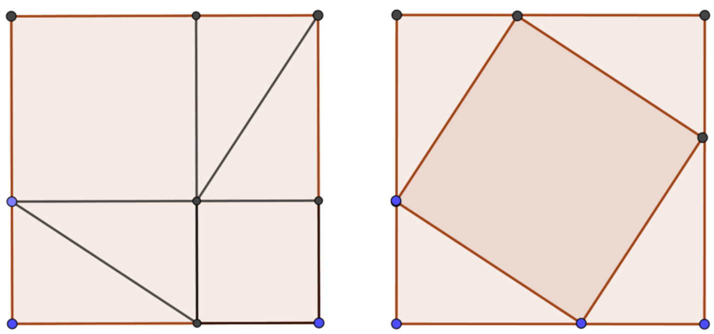
56. Inside a 5-by-5 square, it is possible to place four 3-4-5 triangles so that they do not overlap. Show how. Then explain why you can be sure that it is impossible to squeeze in a fifth triangle of the same size.

57. What do you believe is true about <u>all of the points</u> that are equidistant from two different points?

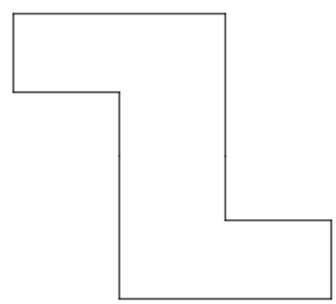
58. Find both points on the line $y = 3$ that are 10 units from the point (2, -3).

59. On a number line where is the number $\dfrac{p+q}{2}$, in relation to p and q?

60. The two-part diagram below, which shows two different dissections of the same square, was designed to help *prove* the Pythagorean Theorem. Provide the missing details.



61. Some terminology: Figures that have exactly the same shape and size are called *congruent*. Dissect the region shown at right into two congruent parts. How many different ways of doing this can you find?



62. Let $A = (2, 4)$, $B = (4, 5)$, $C = (6, 1)$, $T = (7, 3)$, $U = (9, 4)$, and $V = (11, 0)$. Triangles ABC and TUV are specially related to each other. Make calculations to clarify this statement, and write a few words to describe what you discover.

63. A triangle that has two equal sides is called *isosceles*. By giving the coordinates of its vertices, make up an example of an isosceles triangle, one of whose *vertices* is (5, 3). If you can, find a triangle that does not have any horizontal or vertical sides.

64. Let $A = (1, 5)$ and $B = (3, -1)$. Verify that $P = (8, 4)$ is equidistant from A and B. Find at least two more points that are equidistant from A and B. Describe all such points.

65. Go to the [Avenues GeoGebra Book]() and look at the first applet titled *Introducing Congruence*. Find 5 different pairs of congruent triangles by moving the vertices of the two triangles that are shown in the applet. You will know they are congruent when you see the word "true." Take screenshots of your congruent pairs.

66. Find two points on the y-axis that are 9 units from $(7, 5)$.

67. A *lattice point* is a point whose coordinates are *integers*. Find two lattice points that are exactly $\sqrt{13}$ units apart. Is it possible to find lattice points that are $\sqrt{15}$ units apart? Explain your thinking.

68. *Some terminology*: When two angles fit together to form a straight angle (a 180-degree angle, in other words), they are called *supplementary angles*, and either angle is the *supplement* of the other. (So do you think more than two angles can be called supplementary?) When an angle is the same size as its supplement (a 90-degree angle), it is called a *right angle*. When two angles fit together to form a right angle, they are called *complementary angles*, and either angle is the *complement* of the other. Two lines that form a right angle are said to be *perpendicular*. Draw a diagram and give an example for each of these new terms.

69. Can you find two lattice points that are $\sqrt{18}$ units away from each other on the coordinate plane? Explain your thinking.

70. The three angles of a triangle fit together to form a straight angle. In one form or another, this statement is a fundamental *postulate* of Euclidean geometry — accepted as true, without proof. Taking this for granted, then, what can be said about the two non-right angles in a right triangle?

71. Let $P = (a, b)$, $Q = (0, 0)$, and $R = (-b, a)$, where a and b are positive numbers. Show with an argument that angle PQR is right, by introducing two congruent right triangles into your diagram. Verify that the *slope* of *segment QP* is the *opposite sign and the reciprocal* of the slope of segment QR.

72. Find an example of an equilateral hexagon whose sides are all $\sqrt{13}$ units long. Give coordinates for all six points and justify your choices.

73. I have been observing the motion of a bug that is crawling on my graph paper. When I started watching, it was at the point $(1, 2)$. Ten seconds later it was at $(3, 5)$. Another ten seconds later it was at $(5, 8)$. After another ten seconds it was at $(7, 11)$.
(a) Draw a picture that illustrates what is happening. What did you assume?
(b) Where was the bug 25 seconds after I started watching it? What did you assume?
(c) Where was the bug 26 seconds after I started watching it? What did you assume?

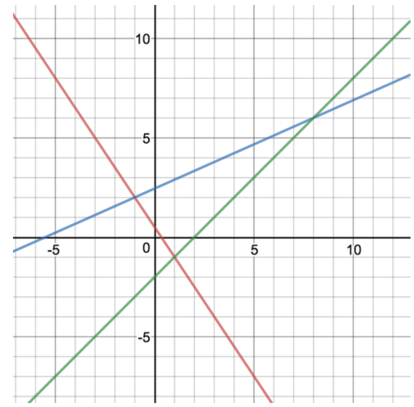
74. The point on segment AB that is equidistant from A and B is called the *midpoint* of AB. For each of the following, find coordinates for the midpoint of AB:
(a) $A = (-1, 5)$ and $B = (3, -7)$ **(b)** $A = (m, n)$ and $B = (k, l)$

75. Is it possible for the $ax + by = c$ to lack a y-intercept? To lack an x-intercept? Explain your thinking.

76. Given the points, $A = (-1,5)$ and $B = (5, 7)$ find a point P that makes APB a right angle. Be prepared to justify your answer.

77. Find the slope of the line through the points given:
(a) $(3, 1)$ and $(3 + 4t, 1 + 3t)$ **(b)** $(m - 5, n)$ and $(5 + m, n^2)$

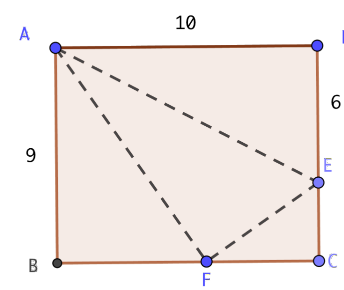
78. As seen in the graph at the right, the sides of a triangle are formed by the graphs of $3x + 2y = 1$, $y = x - 2$, and $-4x + 9y = 22$. Is the triangle isosceles? How do you know?



79. A bug moves linearly with constant speed across my graph paper. I first noticed the bug when it is at (3, 4). It reaches (9, 8) after two seconds and (15, 12) after four seconds.
    (a) Predict the position of the bug after six seconds; after nine seconds; after t seconds.
    (b) Is there a time when the bug is equidistant from the x- and y-axes? If so, where is it?

80. Consider the linear equation $y = 3.62(x - 1.35) + 2.74$.
    (a) What is the slope of this line?
    (b) What is the value of y when $x = 1.35$?
    (c) This equation is written in *point-slope* form. Explain the terminology.
    (d) Use a graphing tool to graph this line.
    (e) Find an equation for the line through $(4.23, -2.58)$ that is *parallel* to this line.
    (f) Use point-slope form to write the equation of a line with a slope of -1.25 and that goes through the point $(-3.75, 8.64)$

81. The dimensions of a rectangular piece of paper $ABCD$ are $AB = 9$ and $BC = 10$. It is folded so that corner D is matched with a point F on edge BC. Given that length $DE = 6$, find EF, EC, FC and the area of triangle EFC.



82. (Continuation) Using the same diagram, it is possible to write the lengths EF, EC, and FC in terms of the length DE. Let $DE= x$, and write the three expressions for these segments.

83. (Continuation) The area of triangle EFC is also describable in terms of DE. Using the expressions you just created for other parts of triangle EFC, write an expression in terms of x for the area of triangle EFC.

84. The x- and y-coordinates of a point are given by the pair of equations $\begin{cases} x = 2 + 2t \\ y = 5 - t \end{cases}$ The coordinates and hence, the position of the point depends on the value assigned to t, known as the *parameter.* Use your graph paper to plot points corresponding to the values $t =$ -4, -3, -2, -1, 0, 1, 2, 3, and 4. Describe any patterns you see.

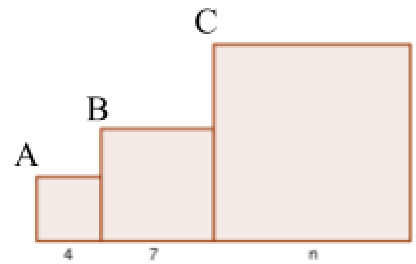
85. A slope can be considered to be a rate. Explain this interpretation.

86. Find a and b so that $ax + by = 1$ has x-intercept 5 and y-intercept 8.

87. Explain the difference between a line that has no slope and a line whose slope is zero.

88. Given that $2x$-$3y$ = 17 and $4x$+$3y$ = 7, and without using paper, pencil, or a calculator, find the value of x. Be prepared to explain your method.

89. Given points A = (2, 7) and B = (3, 3), find two points P that are on the perpendicular bisector of AB. In each case, what can you say about segments PA and PB?



90. Three squares are placed next to each other as shown. The vertices A, B, and C are *collinear*. Find the length n.

91. (Continuation) Using the same diagram, replace the lengths 4 and 7 by m and k, respectively. Express k in terms of m and n.

<u>GeoGebra Activity 2 – Transformations</u>

Go to the *Avenues GeoGebra Book* online and open the Activity titled "Messing with Mona: Introduction to Transformations"

a. Click the box that says "Translate Mona." How was Mona moved? How do you describe this *translation* (which is a type of *transformation*).
b. Under the click boxes there should be a box of yellow text that says "Translate Mona by vector u" – what do you think a vector is? Can you find it in the diagram?
c. What happens when you select the point "Terminal Point" and drag it around the window?
d. Now click the box that says "Rotate Mona." Describe this type of transformation. What is it called?
e. Move the slider that is below the click boxes so that the angle measurement changes. Describe how the image of Mona changes. What is special about point A?
f. Now click the box that says "Reflect Mona About a Line." Describe this transformation. What is it called?
g. Select one of the two yellow points on the line between the two Monas. What happens to the image of Mona when you move the line away from the original Mona Lisa? What happens to the image of Mona when you move the line closer to the original Mona Lisa?

92. A five-foot Avenues student casts a shadow that is 40 feet long while standing 200 feet from a streetlight. How high above the ground is the lamp?

93. (Continuation*)* How far from the streetlamp should the student stand in order to cast a shadow that is long as the student is tall?

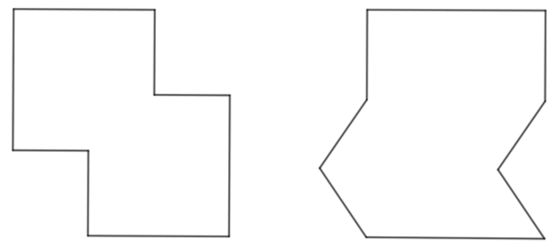
94. An airplane 27,000 feet above the ground begins descending at the rate of 1,500 feet per minute. Assuming the plane continues at the same rate of descent, how long will it be before it is on the ground?

95. (Continuation) Using an appropriate window on a graphing utility, graph the line $y = 27{,}000 - 1500x$. With the preceding problems in mind, explain the significance of the slope of this line and its two intercepts.

96. An airplane is flying at 36,000 feet directly above Lincoln, Nebraska. A little later the plane is flying at 28,000 feet directly above Des Moines, Iowa, which is 160 miles from Lincoln. Assuming a constant rate of descent, predict how far from Des Moines the airplane will be when it lands.

97. In a dream, Blair is confined to a coordinate plane, moving along a line with a constant speed. Blair's position at 4 am is (2, 5) and at 6 am it is (6, 3). What is Blair's position at 8:15 am when the alarm goes off?

98. Find a way to show that points $A = (-4, -1)$, $B = (4, 3)$, and $C = (8, 5)$ are collinear.

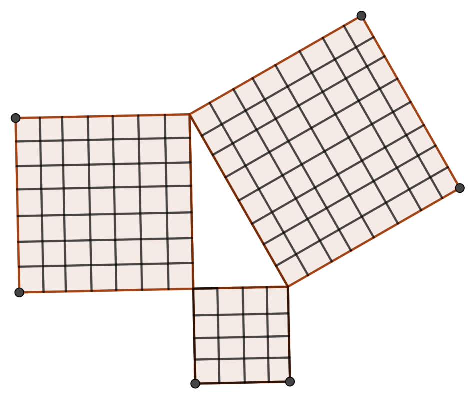


99. Find as many ways as you can to dissect each figure at right into two congruent parts.

100. Write the equation of the line that goes through the point (6, -5) with slope -½:
   a. in point/slope form
   b. in slope/intercept form
   c. in standard form

101. One of the legs of a right triangle is 12 units long. The other leg is b units long and the hypotenuse c units long, where b and c are both integers. Find b and c. Hint: Both sides of the equation $c^2 - b^2 = 144$ can be factored.

102. Is there anything wrong with the figure shown at right?



103. A river bank runs along the line $x = 3$ and a dog is tied to a post at the point $D = (10, 5)$. If the dog's leash is 25 units long (the same units as the coordinates), and if a fence were going to be placed at the edge of the river along $x = 3$, name the two coordinates along the river where it would be safe for the fence to end so that the dog could not fall in the river even though he is tethered at D. How long would the fence be?

104. At noon one day, Corey decided to follow a straight course in a motor boat. After one hour of making no turns and traveling at a steady rate, the boat was 6 miles east and 8 miles north of its point of departure. What was Corey's position at two o'clock? How far had Corey traveled? What was Corey's speed?

105. (Continuation) Assume that the fuel tank initially held 12 gallons, and that the boat gets 4 miles to the gallon. How far did Corey get before running out of fuel? When did this happen? When radioing the Coast Guard for help, how should Corey describe the boat's position?

106. Suppose the numbers a, b, and c represent the sides of a triangle and fit the equation $a^2 + b^2 = c^2$ and $a = b$. Express c in terms of a and draw a diagram of such a triangle. Make a conjecture about such a triangle's angles.

107. Teterboro Airport is 3 km west and 5 km north of Little Ferry, NJ. At 1 pm, Sam took off in a small one-person plane. Every six minutes, the plane's position changed by 9 km east and 7 km north. At 2:30 pm, Sam was flying over the town of New Rochelle, NY. In relation to Little Ferry, (a) What are the coordinates of Teterboro Airport? (b) What are the coordinates of Sam's plane when he is over New Rochelle? (c) What are the coordinates of Sam's plane after t hours of flying?

108. Is it possible to form a square whose area is 34 by connecting four lattice points? Explain.

109. Write a formula for the distance from $A = (-1, 5)$ to $P = (x, y)$, and another formula for the distance from $P = (x, y)$ to $B = (5, 2)$. Then write an equation that says that P is equidistant from A and B. Simplify your equation to linear form (make it look like $y = mx + b$).

110. (Continuation) When you wrote an equation that said that the distance from $A = (-1, 5)$ to $P = (x, y)$ is equal to the distance from $P = (x, y)$ to $B = (5, 2)$, the line you found was called the *perpendicular bisector* of AB. Verify this by calculating two slopes and one midpoint.

111. Golf balls cost \$0.90 each at Andie's Club, which has an annual \$25 membership fee. At Bailey's sporting-goods store, which does not have a membership fee, the price is \$1.35 per ball for the same brand. Where you buy your golf balls depends on how many you wish to buy. Explain, and illustrate your reasoning by drawing a graph.

112. Draw the following segments. What do they have in common? **(a)** from (0,0) to (7, -4)
(b) from (3, -1) to (10,3) **(c)** from (1.3, 0.8) to (8.3, 4.8) **(d)** from $(\pi,\ 4 + \sqrt{2})$ to $(7 + \pi,\ \sqrt{2})$

113. (Continuation) The *directed segments* have the same *length* and some have the same slope and direction. The *components* of those vectors are the numbers 7 and 4.

a. Some directed segments in the problem above represent the *vector* [7, 4], which ones? Find another example of the endpoints of a directed segment that represents the vector [7, 4]
b. What are the components of the two other vectors represented by the directed segments in the problem above?
c. The point at which the segment starts is called the *tail* of the vector, and the final point is called the *head*. Which of the following directed segments represents [7, 4]? In each case, state which point is the tail and which is the head.
    (a) from $A(-2, -3)$ to $B(5, -1)$ **(b)** from $P(-3, -2)$ to $Q(11, 6)$
    (c) from $S(10, 5)$ to $T(3, 1)$ **(d)** from $M(-7, -4)$ to $N(0, 0)$

114. The points (x, y) defined by the equations $x = 1 + 2t$ and $y = 3 + t$ form a line. Decide whether each of the following points are on that line:
(a) (7, 6) **(b)** (-3, 1) **(c)** (6, 5.5) **(d)** (11, 7)

115. The perimeter of an isosceles right triangle is 24. Write an equation that allows you to solve for the lengths of the three sides. Use the graph of that equation to find the solutions.

116. The x- and y-coordinates of a point are given by the equations $\begin{cases} x = -4 + 3t \\ y = 1 + 2t \end{cases}$ Use your graph paper to plot points corresponding to $t = -1$, 0, and 2. These points should appear to be collinear. Convince yourself that this is the case, and calculate the slope of this line. The displayed equations are called *parametric because* of the *parameter, t.* How is the slope of a line determined from its parametric equations?

117. Show that the triangle formed by the lines $y = 2x - 7$, $x + 2y = 16$, and $3x + y = 13$ is isosceles. Show also that the lengths of the sides of this triangle fit the Pythagorean equation. Can you identify the right angle just by looking at the equations?

118. When working with points that have been transformed, "*C* prime" is the usual way of reading *C'* the image of the point *C*. A triangle has vertices $A = (1, 2)$, $B = (3, 5)$, and $C = (6, 1)$. The Image triangle *A'B'C'* is obtained by sliding triangle *ABC* 5 units to the right (in the positive *x*-direction, in other words) and 3 units up (in the positive *y*-direction). It is also customary to say that vector [5, 3] has been used to translate triangle *ABC.* What are the coordinates of the image points *A'*, *B',* and *C'*?

119. (Continuation) When vector [*h, k*] is used to translate triangle *ABC*, it is found that the image of vertex *A* is (-3, 7). What are the images of vertices *B* and *C*?

120. Caught in another nightmare, Blair is moving along the line $y = 3x + 2$. At midnight, Blair's position is (1, 5), the *x*-coordinate increasing by 4 units every hour. Write parametric equations that describe Blair's position *t* hours after midnight. What was Blair's position at 10:15 pm when the nightmare started? Find Blair's speed, in units per hour.

121. An equilateral quadrilateral is called a *rhombus*. A square is a specific example of a rhombus. Find a non-square rhombus whose *diagonals* and sides are *not* parallel to the rulings on your graph paper. Use coordinates to describe its vertices. Write a brief description of the process you used to find your example.

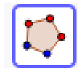
122. Using GeoGebra, plot the points $A = (-5, 0)$, $B = (5, 0)$, and $C = (2, 6)$, then the points $K = (5, -2)$, $L = (13, 4)$, and $M = (7, 7)$. Find the lengths of each side and the measure of each angle of the triangles *ABC* and *KLM.* It is customary to call two triangles *congruent* when all corresponding sides and angles are the same.

123. (Continuation) Are the triangles related by a vector translation? Why?

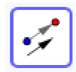
GeoGebra Activity #3 – Vector Translations

Using GeoGebra, you can facilitate visualization of *vector translations*.

A. Open a new GeoGebra file and hide the axes in the Graphics View.

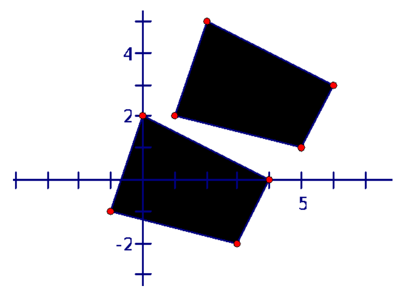
B. Select the *regular* polygon tool  from the polygon toolbox and construct a small regular pentagon.

C. Using the vector tool  draw a vector somewhere else in the Graphics View. This vector is going to serve as your vector for a translation of the regular polygon, so the length of the vector will be how far the polygon is moved and the slope of the vector is the slope it will be moved by. The direction of the vector is denoted by the arrow at its head.

D. Now select the Translate by Vector tool  from the Transformation toolbox and select the polygon and then the vector in order to translate the regular pentagon by the vector you had drawn.

E. Now press escape (to give you back the selection tool) and drag the original vector by the point at the head (arrow) end. What happens to the translated pentagon? Does anything happen to the original pentagon? Why or why not?

124. Let $A = (1, 2)$, $B = (5, 1)$, $C = (6, 3)$, and $D = (2, 5)$. Let $P = (-1, -1)$, $Q = (3, -2)$, $R = (4, 0)$, and $S = (0, 2)$. Use a vector to describe how quadrilateral *ABCD* is related to quadrilateral *PQRS*. What is the length of this vector?



125. Let $K = (3, 8)$, $L = (7, 5)$, and $M = (4, 1)$. Find coordinates for the vertices of the triangle that is obtained by using the vector [2, -5] to slide triangle *KLM*. How *far* does each vertex slide?

126. The *length of a vector* is defined as the hypotenuse of the right triangle created by its *components*. The horizontal component of the vector [-1, 7] is -1 and the vertical component is 7. What is the length of the vector [-1, 7]? What is the length of vector [a, b]? Some notation: the length of a vector is written as $\|[a,b]\|$.

127. Let $A = (2, 4)$, $B = (4, 5)$, and $C = (6, 1)$. Draw three new triangles as follows:

a. ΔPQR has $P = (11, 1)$, $Q = (10, -1)$, and $R = (6, 1)$;

b. ΔKLM has $K = (8, 10)$, $L = (7, 8)$, and $M = (11, 6)$;

c. ΔTUV has $T = (-2, 6)$, $U = (0, 5)$, and $V = (2, 9)$.

These triangles are not obtained from ABC by applying a vector translation. Instead, each of the appropriate transformations is described by one of the suggestive names *reflection*, *rotation*, or *glide-reflection*. Decide which is which and explain your answers.

128. In baseball, the bases are placed at the corners of a square whose sides are 90 feet long. Home plate and second base are at opposite corners. To the nearest tenth, how far is it from home plate to second base in fraction form?

129. Avenues students were given a math project to make a life-size copy of the triangular park at Jackson Square at the corner of 8^{th} Avenue and Greenwich Avenue and build it in Abingdon Square park just down 8^{th} Avenue. The students have a limited amount of time so they measure <u>one of the sides</u> of Jackson Square park and create a congruent segment in Abingdon Square park. If they do not do any more measurements does this guarantee that the new triangle will be congruent to the original one that was Jackson Square? Sketch a diagram of this scenario. Another group measured only one angle and created a congruent angle in Abingdon Square. If they do not do any more measurements does this guarantee that the copy of Jackson Square Park will be congruent to the original? Why or why not? Sketch a diagram.

130. Give the components of the vector whose length is 5 and that points in the opposite direction of [-4, 3].

131. A right triangle has one leg twice as long as the other and the perimeter is 18. Write an equation that allows you to solve for the three sides of this triangle. Use the graph of this equation to solve for the three sides.

132. Let $A = (0, 0)$, $B = (2, -1)$, $C = (-1, 3)$, $P = (8, 2)$, $Q = (10, 3)$, and $R = (5, 3)$. Plot these points. Angles BAC and QPR should look like they are the same size. How would you argue that these angles are the same size without measuring them?

133. Continuing their triangle copying project, the Avenues students realize that copying a single measurement will not guarantee an exact copy of Jackson Square Park. They decide to try measuring <u>two parts</u> (i.e. two angles or one side and one angle, etc.). Are there any combinations of two measurements that ensure that the copy will be congruent to the original?

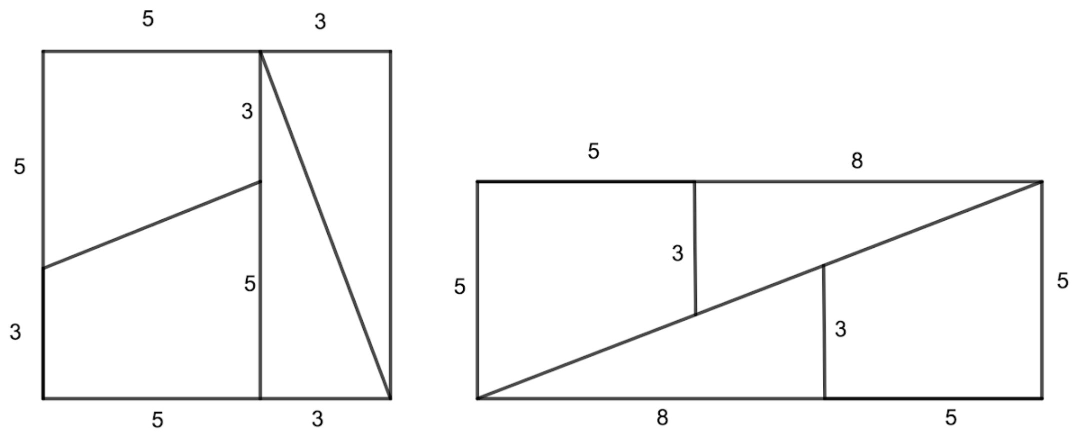
134. What is the slope of the line $ax + by = c$? Find an equation for the line through the origin that is perpendicular to the line $ax + by = c$.

135. A bug is initially at the point (-3, 7). Where is the bug after being displaced by vector [-7, 8]?

136. Instead of saying that Remy moves 3 units left and 2 units up, we know that you can say that Remy's position is displaced by the vector [-3, 2]. Identify the following *displacement vectors*:

(a) Forrest starts at (2, 3) at 1 pm, and has moved to (5, 9) by 6 am the next morning;
(b) at noon, Eugene is at (3, 4); two hours earlier Eugene was at (6, 2);
(c) during a single hour, a small airplane flew 40 miles north and 100 miles west.

137. Compare the two figures shown below. Is there anything wrong with what you see? Write a few sentences justifying your answer.



138. Let $M = (a, b)$, $N = (c, d)$, $M' = (a + 2, b + 3)$, and $N' = (c + 2, d + 3)$. Use the distance formula to show that segments MN and $M'N'$ have the same length. Explain why this could be expected. (The points M and N could be anywhere so feel free to put them in the first quadrant).

139. State the components of a vector that points in the opposite direction of [3,4] and is twice as long.

140. Tracy and Kelly are running laps on an indoor track – at steady speeds, but in opposite directions. They meet every 20 seconds. It takes Tracy 45 seconds to complete each lap. How many seconds does it take for each of Kelly's lap? Check your answer.

141. When working with points defined parametrically, the notation, P_t, and P_2 are usually read "P sub t" and "P sub 2" indicating the t value at that point. The initial position of an object is $P_0 = (7, -2)$ (when $t = 0$). Its position after being displaced by the vector $t[-8,7]$ is $P_t = (7, -2) + t[-8, 7]$. Notice that the meaning of "+" is to apply a vector translation to P_0. Notice also that the position is a function of t. Calculate P_3, P_2, and P_{-2}. Describe the configuration of all possible positions P_t.

GeoGebra Activity #4 – Triangle Congruence Criteria

A. In the *Avenues: The World School GeoGebra Book of Resources* Chapter on Triangle Congruence Applets for Math 2. The class has determined that two parts are not enough to show congruence between two triangles. So that leaves us with combinations of three parts.
B. Your task is to play with each of these applets and decide which of the 3-part combinations are, in fact, true congruence criteria. This means that these three parts are enough to show that two triangles are always congruent. What will be your decision-making strategy? How will you know if three parts of one triangle being congruent to three parts of another triangle makes every single part of the two triangles congruent to each other?
C. Make a table like the one below so that you have clear conclusions about each combination.

3-Part Combination	Is it always enough to guarantee congruence between 2 triangles?	Why?

142. Two of the sides of a right triangle have length $360\sqrt{2018}$ and $480\sqrt{2018}$. Find the possible lengths of the third side.

143. If I were to increase the length of my stride by one inch, it would take me 60 fewer strides to cover a mile. What was the length of my original stride?

144. Let $A = (1, 4)$, $B = (0, -9)$, $C = (7, 2)$ and $D = (6, 9)$. Prove that the triangles *DAB* and *DCB* are congruent. Be sure to use one of the triangle congruence criteria.

145. Plot points $K = (0, 0)$, $L = (7, -1)$, $M = (9, 3)$, $P = (6, 7)$, $Q = (10, 5)$, and $R = (1, 2)$. Show that the triangles *KLM* and *RPQ* are congruent. Which triangle congruence criteria did you use?

146. (Continuation) Is *KLM* or *RPQ* a vector translation of each other? If not, describe how one triangle has been transformed into the other.

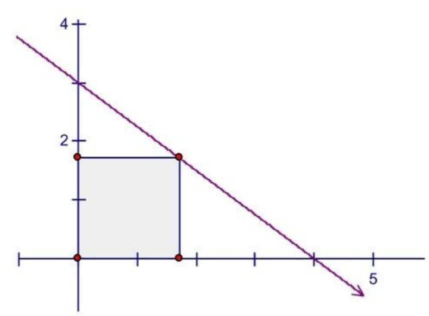
147. (Continuation) If two figures are congruent, then their parts *correspond.* In other words, each part of one figure has been matched with a definite part of the other figure. In the triangle *PQR*, which angle corresponds to angle *M*? Which side corresponds to *KL*? In general, what can be said about corresponding parts of congruent figures? How might you confirm your hunch experimentally?

148. A nice acronym to shorten the statement about corresponding parts of congruent triangles can be written as CPCTC. What sentence do you think these letters represent?

149. Given the vector [-5,12], find the following vectors:

(a) Same direction, twice as long
(b) Same direction, length 1
(c) Opposite direction, length 10
(d) Opposite direction, length c

150. The diagram at the right shows the graph of $3x + 4y = 12$. The shaded figure is a square, three of whose vertices are on the coordinate axes. The fourth is on the line. Find:
(a) the x- and y-intercepts of the line.
(b) the length of a side of the square
(c) Show that the vertex that lies on the line is equidistant from the coordinate axes.



151. Blair is in another dream on the coordinate plane and is walking along the line $y = 3x - 2$. A bug starts walking towards Blair perpendicularly from the point (5, 3). What is the equation of the line that describes the bug's path?

152. Plot the three points P = (1, 3), Q = (5, 6), and R = (11.4, 10.8). Verify that PQ = 5, QR = 8, and PR = 13. What is special about these points?

153. *Some terminology:* When the components of the vector [5,-7] are multiplied by a given number t, the result may be written either as [5t, -7t] or as t[5,-7]. This is called the *scalar multiple* of vector [5,-7]. (Another way to think about it, is that it has been scaled by a factor of t). Find the components of the following scalar multiples:

(a) [12,3] by 5
(b) $\left[\sqrt{5}, \sqrt{10}\right]$ by $\sqrt{5}$
(c) $\left[\frac{3}{4}, \frac{2}{3}\right]$ by $\left(-\frac{1}{2} + \frac{2}{3}\right)$
(d) $[p, q]$ by $\frac{p}{q}$

154. Sidney calculated three distances of the collinear points *A, B*, and *C*. The calculations were *AB*= 29, *BC* = 23 and *AC* =54. What do you think of Sidney's data, and why?

155. Find the number that is 2/3 of the way from (a) -7 to 17 (b) m to n

156. The diagonal of a rectangle is 15 cm and the perimeter is 38, what is the area? It is possible to find the answer without finding the length and width of the rectangle – try it.

157. After drawing the line $y = 2x - 1$ and marking the point $A = (-2, 7)$, Kendall is trying to decide which point on the line is closest to A. The point $P = (3, 5)$ looks promising. To check that P is really the point that is closest to A, what would help Kendall decide? Is P closest to A?

158. Let $A = (2, 9)$, $B = (6, 2)$ and $C = (10, 10)$. Verify with your own work that segments AB and AC have the same length. Use GeoGebra to measure angles ABC and ACB. Propose a general statement that applies to any triangle that has two sides of equal length.

159. Find the lengths of the following vectors
(a) [3,4] (b) 2018 [3,4] (c) $\frac{2018}{5}$ [3,4] (d). -2[3,4] (e) t[3,4] (f) $t[a, b]$

160. Given the points $K = (2, -1)$ and $M = (3, 4)$, find coordinates for a point J that makes angle JKM a right angle.

161. Find a point on the line $2x + y = 8$ that is equidistant from the coordinate axes. How many such points are there?

162. Two frogs are trying to jump across a small river. The first frog says, "I only need one stone placed right in the middle to jump from the first riverbank to the second." The second frog says, "I don't think I can do that, I'll probably need two to be placed evenly in order to get to the other side." If they are jumping from the point $A = (2, -1)$ along the vector [6, 12]. Where does the first frog place its stone? Where does the second frog place its stones?

163. When two lines intersect, four angles are formed. It is not hard to believe that the nonadjacent angles in this arrangement are congruent. If you had to prove this to a skeptic, what reasons would you offer? These pairs of angles are called *vertical angles*.

164. A line goes through the points (2, 5) and (6, -1). Let P be the point on this line that is closest to the origin. Calculate the coordinates of P.

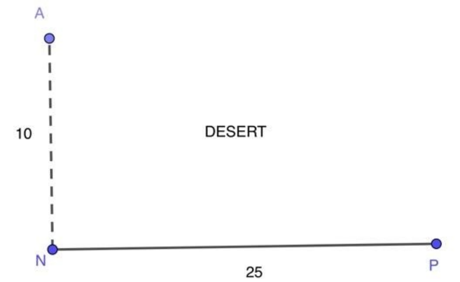
165. Let $K = (-2, 1)$ and $M = (3, 4)$. Find two points that divide KM into three equal parts.

166. We have conjectured that in an Isosceles Triangle the angles opposite the congruent sides seem to always be congruent. Write an argument supporting this assertion, which might be called the *Isosceles Triangle Theorem.*

167. What are the possible criteria to prove that two triangles are congruent?

168. The components of vector [24, 7] are 24 and 7. Find the components of a vector that is three fifths as long as [24,7].

169. Alex the geologist is in the desert, 10 km from a long, straight road. On the road, Alex's jeep can do 50 kph, but in the desert sands, it can manage only 30 kph, Alex is very thirsty, and knows that there is a gas station 25 km down the road (from the nearest point *N* on the road) that has ice-cold Pepsi.

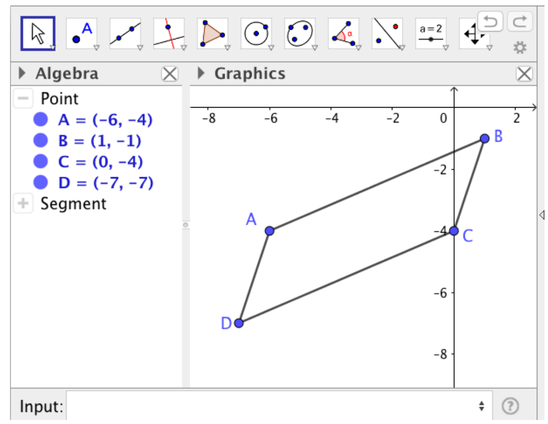


(a) How many minutes will it take for Alex to drive to *P* through the desert?

(b) Would it be faster if Alex first drove to *N* and then used the road to get to *P*?

(c) Find an even faster route for Alex to follow. Is your route the fastest possible? How do you know?

170. Let quadrilateral *ABCD* be defined by the points in the given diagram. Show that the opposite sides of the quadrilateral are parallel. Such a quadrilateral is called a *parallelogram*.



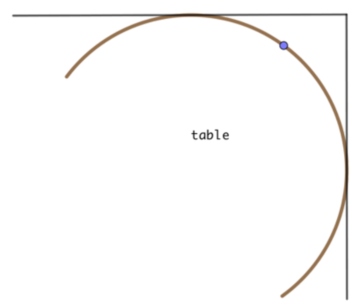
171. What is the number midway between $24-\sqrt{3}$ and $24+\sqrt{3}$? What is number midway between $\frac{-b-\sqrt{b^2-4ac}}{2a}$ and $\frac{-b+\sqrt{b^2-4ac}}{2a}$?

172. If a line intersects the *x*-axis at $(a, 0)$ and intersects the *y*-axis at $(0, b)$ at what point does it intersect the line $y = x$?

173. If *M* is the midpoint of the segment *AB*, how are vectors $\vec{AM}$, $\vec{AB}$, $\vec{MB}$ and $\vec{BM}$ related to each other by direction and size?

174. Show that the lines $3x-4y=-8$, $x=0$, $3x-4y=12$, and $x=4$ form the sides of a rhombus.

175. A circular seminar table is placed in a corner of a room so that it touches both walls. A mark is made on the edge of the table, exactly 18 inches from one wall and 25 inches from the other. What is the radius of the table?



176. Given the points *D, A*, and *Y* with the property that *DA* = 5, *AY* = 7, and *DY* = 12, what can be said about these three points? What would be true if *DY* is less than 12?

177. Given *A* = (-1, 5), *B* = (*x*, 2) and *C* = (4, -6) and the sum of *AB*+*BC* is to be as small as possible, find the value of *x*.

178. Let *A* = (0, 0), *B* = (4, 2) and *C* = (1, 3), find the size of angle *CAB*. Justify your answer.

179. Let $A = (3, 2)$, $B = (1, 5)$ and $P = (x, y)$. Find x and y values that make ABP a right angle.

180. (Continuation) Describe the configuration of all such points P.

181. Find the coordinates for the vertices of a lattice rectangle that is three times as long as it is wide, with none of the sides horizontal.

182. Describe a transformation that carries the triangle with vertices (0, 0), (13, 0) and (3, 2) onto the triangle with vertices (0, 0), (12, 5) and (2, 3). What evidence do you have of the type of transformation that you claim? Where do you think this transformation takes the point (6, 0)? If you cannot find the exact coordinates, make your best guess.

183. Suppose that triangle ACT has been shown congruent to triangle ION, with vertices A, C and T corresponding to vertices I, O and N respectively. It is customary to record result by writing $\Delta ACT \cong \Delta ION$. Notice that corresponding vertices occupy corresponding positions in the congruence statement or "equation." Let $B = (5, 2)$, $A = (-1, 3)$, $G = (-5, -2)$, $E = (1, -3)$ and $L = (0, 0)$. Using only these five labels, find as many pairs of congruent triangles as you can, and express the congruences accurately.

184. (Continuation) How many ways are there of arranging the six letters of $\Delta ACT \cong \Delta ION$ and express the two-triangle congruence correctly?

185. What can be concluded about triangle ABC if it is given that
(a) $\Delta ABC \cong \Delta BCA$ (b) $\Delta ABC \cong \Delta ACB$ (two separate situations)

186. Plot points $K = (-4, -3)$, $L = (-3, 4)$, $M = (-6, 3)$, $X = (0, -5)$, $Y = (6, -3)$, and $Z = (5, 0)$. Show that triangle KLM is congruent to triangle XZY. Describe a transformation that transforms KLM onto XZY. Can you find the equation of a line that helps describe this transformation?

187. Using a triangle congruence criteria, write a proof that the two acute angles in a right triangle are complementary.

188. Alex the geologist is in the desert, 10 km from the nearest point N on a long, straight road. Alex's jeep can do 50 kph on the road, and 30 kph in the desert. Find the shortest time for Alex to reach an oasis that is on the road (a) 20 km from N; (b) 30 km from N.

189. In GeoGebra, draw a triangle, and using the measure tool, measure the lengths of the three sides. Make a conjecture about what must be true about the two sides in comparison to the third side in order for the triangle to exist.

190. Let $A = (0, 0)$, $B = (1, 2)$, $C = (6, 2)$, $D = (2, -1)$ and $E = (1, -3)$. Show that angle CAB is the same size as angle EAD. You may want to use GeoGebra to help you solve this problem.

191. What is true about all of the points that are on the perpendicular bisector of a segment?

192. Is it possible for a line to go through (a) no lattice points? (b) exactly one lattice point? (c) exactly two lattice points? For each answer, either give an example or else explain the impossibility.

193. Given that $A = (6, 1)$, $B = (1, 3)$, $C = (4, 3)$, find a lattice point P that makes $\vec{CP}$ perpendicular to $\vec{AB}$.

194. (Continuation) Describe the set of all points P for which $\vec{AB}$ and $\vec{CP}$ are perpendicular.

195. In quadrilateral *ABCD*, it is given that $AB = CD$ and $BC = DA$. Write an argument as to why the angles *ACD* and *CAB* must be the same size. Be sure to look for congruent triangles. *Note*: If a polygon has more than three vertices the *labeling convention* is to place the letters around the polygon in the order that they are listed. Thus *AC* should not be one of the sides of the quadrilateral here.

196. *The Triangle Inequality Theorem*: What must be true about the three sides of a triangle for it to exist?

197. Find a point on the line $x + 2y = 8$ that is equidistant from (3, 8) and (9, 6).

198. Make up a geometry problem whose answer can be found with the equation $x + 3x + x\sqrt{10} = 42$.

199. How large a square can be put inside a right triangle whose legs are 5 cm and 12 cm?

200. You are one mile from the railroad station and your train is due to leave in ten minutes. You have been walking at a steady rate of 3 mph, and you can run at 8 mph if you have to. For how many more minutes can you continue walking, until it becomes necessary for you to run the rest of the way to the station?

201. Let $A = (-2, 3)$, $B = (6, 7)$ and $C = (-1, 6)$.
(a) Find an equation of the perpendicular bisector of *AB*
(b) Find an equation for the perpendicular bisector of *BC*
(c) Find coordinates of the point *K* that is equidistant from *A*, *B*, and *C*.

202. Given $P = (3, 2)$, $Q = (-4, -4)$ and $R = (6, -4)$. Find the area of triangle *PQR* by two different methods.

203. What argument would you use to prove that the diagonal of a square creates two congruent triangles? Show how.

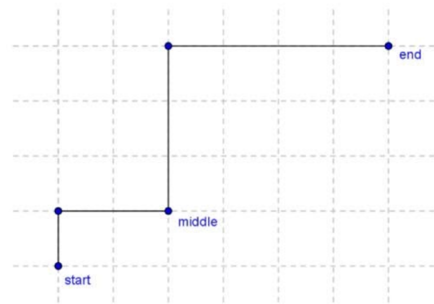
GeoGebra Activity #5 – Altitudes

Go to the *Avenues GeoGebra Book Online* and open the file titled *"Exploring Altitudes"*

(a) You should notice that there is a point of concurrency (intersection) of all of the altitude of the triangle. What is that named?
(b) Choose point B or C and drag it around the window. Try to create all different triangles. Do all of the altitudes remain concurrent?
(c) What do you observe when the triangle is obtuse about the location of the orthocenter?

(d) What do you observe when the triangle is acute about the location of the orthocenter?
(e) What do you observe when the triangle is right? Make measurements to justify your claim for this.

204. Terry walked one mile due north, two miles due east, then three miles due north again and then once more east for 4 miles. How far is Terry from the starting point? Which is farther – Terry's distance from the starting point or the sum of the two direct distances walked?



205. An *altitude* of a triangle is a segment that joins one of the three vertices to a point on the line that contains the opposite side, the intersection being perpendicular. For example, consider the triangle whose vertices are $A = (0,0)$, $B = (8,0)$ and $C = (4,12)$.

a. Find the length of the altitude from C to side AB.
b. Find an equation for the line that contains the altitude from A to side BC.
c. Find an equation for the line BC.
d. Find coordinates for the point where the altitude from A meets side BC.
e. Find the length of the altitude from A to side BC.
f. This should not be the first time you are using altitudes (or heights) of triangles in mathematics. As a check on your work so far, calculate the length of BC and multiply it by $\frac{1}{2}$ of your answer to part (e). You should be able to tell if your answer is correct.
g. It is possible to deduce the length of the altitude from B to side AC from what you have already calculated. Show how.

206. The segments that connect opposite vertices of a quadrilateral are called the *diagonals* of the quadrilateral. The diagonals AC and BD of quadrilateral $ABCD$ intersect at point O. Given the information that $AO = BO$ and $CO = DO$, (but not all four are equal), What can you deduce about the lengths of the sides of $ABCD$? Be sure to have a solid argument for your statement.

207. True or false: $\sqrt{4x} + \sqrt{9x} = \sqrt{13x}$

208. Find the area of the triangle defined by $E = (-2, 8)$, $W = (11, 2)$, and $S = (-2, -4)$. Now find the area of triangle WLS where L is $(-2, 0)$.

209. Maintaining constant speed and direction for an hour, Whitney travelled from (-2, 3) to (10, 8). Where was Whitney after 35 minutes? What distance did Whitney cover in those 35 minutes?

210. True or False: If a quadrilateral is equilateral, its diagonals are perpendicular. Explain.

211. The line $3x + 2y = 16$ is the perpendicular bisector of the segment AB. Find coordinates of point B, given that **(a)** $A = (-1, 3)$ **(b)** $A = (0, 3)$

212. (Continuation) Point B is called the *reflection A* across the line $3x + 2y = 16$. Sometimes B is simply called the image of A. Using the same line, find another point C and its image C'. Explain a method for finding the reflection of a point over a line.

213. Triangle ABC is isosceles with $AB = BC$, and angle BAC is 56 degrees. Find the remaining two angles of this triangle.

214. Let $A = (-4, 0)$, $B = (0, 6)$ and $C = (6, 0)$. Find the coordinates of the midpoint of AC. Write the equation of the line segment that connects B to the midpoint of AC.

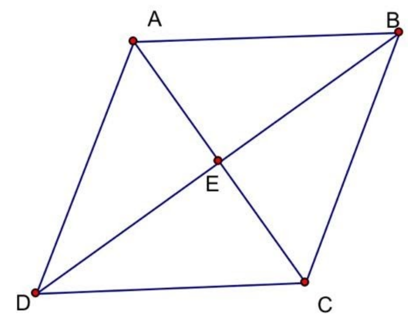
215. Find the area of the triangle whose vertices are (-2, 3), (6, 7) and (0, 6).

216. Pat and Chris were out in their rowboat one day and Chris spied a water lily. Knowing that Pat likes a mathematical challenge, Chris announced that, with the help of the plant, it was possible to calculate the depth of the water under the boat. When pulled taut, directly over its root, the top of the plant was originally 10 inches above the water's surface. While Pat held the top of the plant, which remained rooted to the lake bottom, Chris gently rowed the boat five feet. This forced Pat's hand toward the water's surface until it was touching. Use this information to calculate the depth of the water.

217. Triangle ABC is isosceles with $AB = BC$, and angle ABC is 56 degrees. Find the remaining two angles of this triangle.

Fill in the blanks to complete the proof logically.

1. Prove that <u>in a rhombus</u>, the diagonals create <u>four congruent triangles</u>.



We know that $AB \cong BC \cong CD \cong DA$ because ______________________.

Since points *B* and *D* are equidistant from *A* and *C*, segment *BD* must be the

____ of *AC*. Since *A* is

equidistant from *B* and *D*, segment *AC* must be the

____ of *BD*. Since *AC* and

BD perpendicularly bisect each other, the point *E* must be the midpoint of ___________and ___________.

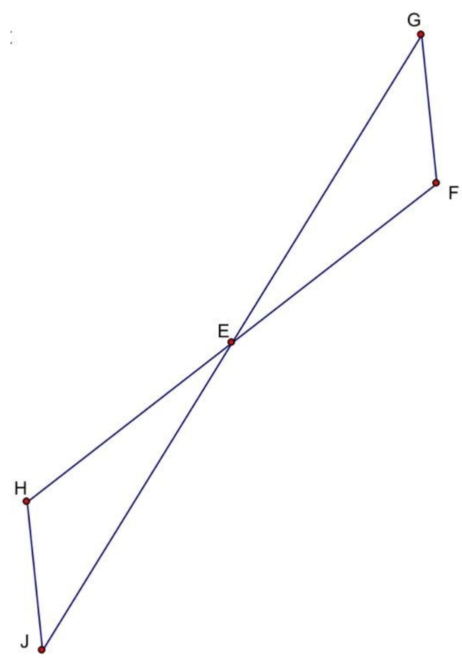
Since *E* is the midpoint of both *AC* and *BD*, we can say that segments ________ ≅ ________ and also,

________ ≅ ________. Finally, we can say that because of the SSS triangle congruence criteria, the

triangles ________ ≅ _______ ≅ ________ ≅ ________. Therefore, the diagonals create four congruent

triangles in a rhombus.

2. Given the following diagram, and that *HF* and *JG* bisect each other at point *E*, prove that <u>$\angle H$ and $\angle F$ are congruent.</u>



Since *HF* and *JG* bisect each other at point *E*, we can say that

________ ≅ ________ and ________ ≅ ________. We can also say that the

pair of angles ∠ ____________ and ∠ ___________ are congruent to

each other because they are vertical angles. Therefore, by

_________________, we can say that triangles ________ ≅ _______.

In conclusion, $\angle H \cong \angle F$ because of _______________________.

218. What is true about the diagonals of a rhombus? Justify your statement with a proof. Start with what you know is true about any rhombus *ABCD* from the definition of a rhombus.

219. Let $A = (1, 4)$, $B = (8, 0)$ and $C = (7, 8)$. Find the area of triangle *ABC.*

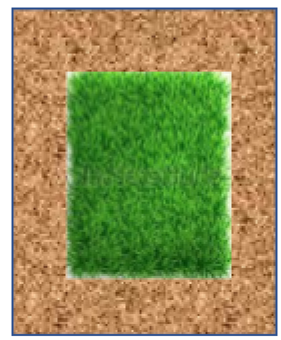
220. A rhombus has 25 cm sides, and one diagonal is 14 cm long. How long is the other diagonal?

221. Write a proof for this statement: In a kite, one of the diagonals is bisected by the other. Start by using the definition of a kite, to state what you know about the quadrilateral *ABCD* which is a kite.

222. If *ABC* is a right triangle with *B t*he right angle, $A = (-3, 2)$, $B = (2, 5)$, find possible coordinates for *C*.

223. Prove that one of the diagonals of a kite bisects two of its angles and the other does not. Again, start with a general kite *ABCD* and what must be true about that kite. If a point is equidistant from the endpoints of a segment where must it lie? How is this helpful in this proof?

224. Robin is mowing a rectangular field that measures 24 meters by 32 meters, by pushing the mower around and around the outside of the plot. This creates a widening border that surrounds the unmowed grass in the center. During a brief rest, Robin wonders whether the job is half done yet. How wide is the uniform border when the job is half done? Use the graph of the equation you have found to confirm your answer.



GeoGebra Activity #6– Medians

Go to the [Avenues GeoGebra Book]() online and find the activity titled "*Exploring Medians and the Centroid*."

a. What can be said about the three medians of a triangle?
b. Do the properties that you observed for the locations of the orthocenter hold true for centroid? Test your conjecture by making the triangle right, acute and obtuse. What is your reasoning for why it does or does not hold.
c. Click the show/hide buttons for the different parts of each of the medians? Try to make a conjecture about where the centroid lies on each median by looking at the lengths of each segment. Discuss with your classmates to see if your conjecture agrees with others.

225. Suppose that the vectors $[a, b]$ and $[c, d]$ are perpendicular. Show that $ac + bd = 0$.

226. Let $A = (0, 0)$, $B = (4, 3)$, $C = (2, 4)$, $P = (0, 4)$, and $Q = (-2, 4)$. Decide whether angles *BAC* and *PAQ* are the same size (congruent, that is), and give your reasons. GeoGebra may be helpful here.

227. If a triangle is Isosceles, then the medians drawn to the congruent sides must have the same length. Prove this statement.

228. Let $A = (0, 12)$ and $B = (25, 12)$. If possible, find coordinates for a point P on the x-axis that makes APB a right angle.

229. Given points $A = (0, 0)$ and $B = (-2, 7)$, find coordinates for points C and D so that $ABCD$ is a square.

230. Let $A = (-4, 0)$, $B = (0, 6)$, and $C = (6, 0)$.
(a) Find equations for the three medians of triangle ABC.
(b) Show that the three medians are concurrent, by finding coordinates for their common point. The point of concurrence is called the *centroid* of triangle ABC.

231. Find k so that the vectors [4, -3] and [k, -6] are
(a) parallel (b) perpendicular

232. The lines $3x + 4y = 12$ and $3x + 4y = 72$ are parallel. Explain why. Find the distance that separates these lines. You will have to decide what "distance" means in this context.

233. Give an example of an equiangular polygon that is not equilateral.

234. Find coordinates for a point on the line $4y = 3x$ that is 8 units from (0,0).

235. Is it possible for a rectangle to have a perimeter of 100 feet *and* an area of 100 square feet? Justify your response.

236. On a separate sheet of paper, draw parallelogram $PQRS$ with vertices at P= (0,0), Q= (3,6), R= (9, 8) and S= (6,2). Cut out your parallelogram {REALLY, CUT IT OUT}. By making one other cut through the parallelogram, show how a rectangle can be formed by the two pieces you have. What can you conclude about the area of a parallelogram from this demonstration?

237. Let $A = (0, 0)$, $B = (8,1)$, $C = (5, -5)$, $P = (0, 3)$, $Q = (7, 7)$, and $R = (1, 10)$. Prove that angles ABC and PQR have the same size.

238. (Continuation) Let D be the point on segment AB that is exactly 3 units from B, and let T be the point on segment PQ that is exactly 3 units from Q. What evidence can you give for the congruence of triangles BCD and QRT?

239. Prove that if a triangle is isosceles, then the altitudes drawn to the congruent sides must be congruent. Begin by drawing the altitudes BN and AM to the congruent sides AC and BC of isosceles triangle ABC. Find two triangles that might be congruent of which BN and AM might be corresponding parts.

240. Find the area of a parallelogram with coordinates (1, 3), (7, 3), (5, 6) and (11, 6). Justify your answer.

241. Give the points $A = (0, 0)$, $B = (7, 1)$, and $D = (3, 4)$, find coordinates for the point C that makes quadrilateral $ABCD$ a parallelogram. What if the question had requested $ACBD$ instead?

242. In triangle ABC, it is given that $CA = CB$. Points P and Q are marked on segments CA and CB respectively so that angles CBP and CAQ are the same size. Prove that $CP = CQ$.

243. Plot all the points that are 3 units from the x-axis. Describe the configuration algebraically.

244. Let $A = (-4, 0)$, $B = (0, 6)$, and $C = (6, 0)$.
(a) Find equations for the three lines that contain the altitudes of triangle ABC.
(b) Show that the three altitudes are concurrent, by finding coordinates for their common point. The point of concurrence is called the *orthocenter* of triangle ABC.

245. The equation $y - 5 = m(x - 2)$ represents a line, no matter what value m has.
   a. What do all these lines have in common?
   b. When $m = -2$, what are the x- and y-intercepts of the line?
   c. When $m = -1/3$, what are the x- and y-intercepts of the line?
   d. When $m = 2$, what are the x- and y-intercepts of the line?
   e. For what values of m are the axis intercepts both positive?

246. Find the area of a triangle with vertices at (-3, -2), (3, -1) and (8, 6).

247. *Some terminology:* Draw a parallelogram whose adjacent edges are determined by vectors [2, 5] and [7, -1], placed so that they have a common initial point. This is called placing the vectors *tail-to-tail*. Find the area of this parallelogram.

248. Asked to reflect the point $P = (4, 0)$ across the mirror line $y = 2x$, Aubrey reasoned this way, "First mark the point $A = (1, 2)$ on the line, then use the vector [-3, 2] to translate P to A then again from A to $P' = (-2,4)$, which is the requested image point." What did Aubrey do wrong? Explain.

249. A polygon that is both equilateral and equiangular is called *regular*. Prove that all diagonals of a regular pentagon (five sides) have the same length.

250. Find the point on the y-axis that is equidistant from $A = (0,0)$ and $B = (12, 5)$.

251. Let $P = (-1, 3)$. Find the point Q for which the line $2x + y = 5$ serves as the perpendicular bisector of segment PQ.

252. The equation $y - 5 = m(x - 2)$ represents a line, no matter what value m has.
   a. What are the x- and y-intercepts of this line?
   b. For what value of m does this line form a triangle of area 36 with the positive axes?
   c. Show that the area of a first-quadrant triangle formed by this line must be at least 20.

GeoGebra Activity #7 – Perpendicular Bisectors

Go to the *Avenues GeoGebra Book* online and find the activity titled "*Perpendicular Bisectors of a Triangle*."

a. Select any vertex of the triangle and move it around. Observe what happens to the *point of concurrency* of the perpendicular bisectors (*circumcenter*). What happens to this point when the triangle is right? obtuse? acute?

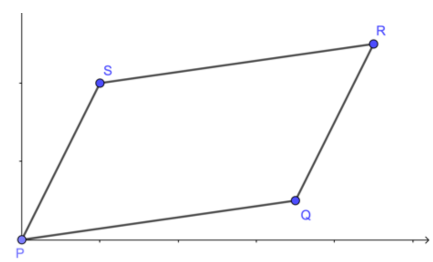
b. Why might the circumcenter and the orthocenter behave in the same ways with regards to its location depending on what type of triangle you have?

c. The circumcenter also has another interesting property. Recall the property of perpendicular bisectors discussed in class. The intersection of the perpendicular bisectors then has that property for both segments (in this case all three). Discuss with some classmates what you think might be true about the circumcenter.

253. Let $A = (3, 4)$, $B = (0, -5)$, and $C = (4, -3)$. Find equations for the perpendicular bisectors of the segments AB and BC.
   a. Find the coordinates for their intersection point K.
   b. Calculate the lengths KA, KB and KC.
   c. Why is K also on the perpendicular bisector of segment CA?

254. (*Continuation*) A *circle* centered at K can be drawn so that it goes through all three vertices of triangle ABC. Explain why. This is why K is called the circumcenter of the triangle. In general, how do you locate the circumcenter of a triangle?

255. The figure at right shows a parallelogram $PQRS$, three of whose vertices are $P = (0,0)$, $Q = (a, b)$ and $S = (c, d)$.
   a. Find the coordinates of R.
   b. Find the area of $PQRS$, and simplify your formula.



256. Find points on the line 3x +5y = 15 that are equidistant from the coordinate axes.

257. Find coordinates for the point that is equidistant from (-1, 5), (8, 2) and (6, -2). How do you know there is one?

258. Find an equation for the line through point (7, 9) that is perpendicular to the vector [5, -2].

259. Find an equation for the line that goes through (5, 2) and that forms a triangle in the first quadrant that is just large enough to enclose the 4-by-4 square in the first quadrant that has two of its sides on the coordinate axes.

260. Describe a transformation that carries the triangle with vertices (1, 2), (6, 7) and (10, 5) onto the triangle with vertices (0, 0), (7, -1), and (9, 3). Where does your transformation send (7, 4)?

261. Find the area of the parallelogram whose vertices are (2, 5), (7, 6), (10, 10), and (5, 9).

262. Let A=(1, 3), B=(7, 5), C=(5, 9). Answer the item below that is determined by the first letter of your last name. Algebraically find coordinates for the requested point.
(a-e) Show that the three medians are concurrent at a point G.
(f-m) Show that the three altitudes are concurrent at a point H.
(n-z) Show that the perpendicular bisectors of the sides of ABC are concurrent at a point K. What special property does K have?

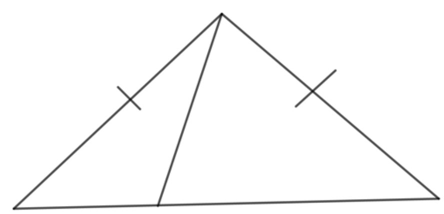
263. Find the coordinates of a point that is three times as far from the origin as (2,3) is. Describe the configuration of all such points.

264. A triangle that has a 13-inch side, a 14-inch side, and a 15-inch side has an area of 84 square inches. Accepting this fact, find the lengths of all three altitudes of this triangle.

265. Simplify the equation algebraically $\sqrt{(x-3)^2+(y-5)^2}=\sqrt{(x-7)^2+(y+1)^2}$. Interpret your result.

266. Given that $ABCDEFGHI$ is a regular polygon, prove the AD and FI have the same length.

267. Use the diagram to help you explain why SSA evidence is not by itself sufficient to justify the congruence of triangles. The tick marks designate segments that have the same length.



268. Find the lengths of all of the altitudes of a triangle whose vertices are (0, 0), (3, 0) and (1, 4).

269. The *converse* of a statement of the form "If A then B" is the statement "If B then A." Write the converse of the statement "If it is Fifth Term, we have math class." Consider the statement "During Fifth Term, we have math class." Is that equivalent to the original statement or the converse?

270. (Continuation) "If point P is equidistant from the coordinate axes, then point P is on the line y=x." Is this a true statement?
   a. Write the converse of this statement. Is it true?
   b. Give an example of a true statement whose converse is false.
   c. Give an example of a true statement whose converse is true.

271. The diagonals of a kite are 6 cm and 12 cm long. Is it possible for the lengths of sides of this kite to be in a 2-to-1 ratio?

272. You have recently seen that there is no completely reliable SSA criterion for congruence. If the angle part of such a correspondence is a *right* angle, however, the criterion is reliable. Justify this so-called *hypotenuse-leg* criterion (which is abbreviated HL).

273. It is given that $a + b = 6$ and $ab = 7$.

a. Find the value of $a^2 + b^2$. Can you do this without finding values for a and b?

b. Make up a geometry problem that corresponds to the question in part (a).

274. Suppose that triangle PAB is isosceles, with $AP = PB$, and that C is on side PB, between P and B. Show that $CB < AC$,

275. Find the area of a triangle that has sides 10, 10, and 5.

276. Let $P = (2, 7)$, $B = (6, 11)$ and $M = (5, 2)$. Find a point D that makes $\vec{PB} = \vec{DM}$. What can you say about quadrilateral $PBMD$?

277. Given that (-1, 4) is the reflected image of (5, 2), find an equation for the line of reflection.

278. The diagonals of quadrilateral $ABCD$ intersect perpendicularly at O. What can be said about quadrilateral $ABCD$?

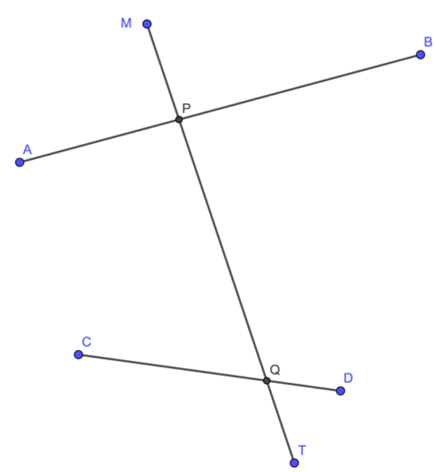
279. A stop sign – a regular octagon – can be formed from a 12-inch square sheet of metal by making four straight cuts that snip off the corners. How long, to the nearest 0.01 inch, are the sides of the resulting polygon?

280. What do you call **(a)** an equiangular quadrilateral? **(b)** an equilateral quadrilateral **(c)** a regular quadrilateral?

281. The diagram at right shows lines APB and CQD intersected by line $MPQT$ which is called a *transversal.* There are two groups of angles formed: one group of four angles with vertex at P and the other group at vertex Q. There is special terminology to describe pairs of angles in this scenario – one from each group at the different vertices.

If the angles are on different sides of the transversal, they are called *alternate*, for example angles APM and PQD. Angle BPQ is an *interior* angle because it is between the lines AB and CD. Thus, angles APQ and PQD are called *alternate interior*, while angles QPB and PQD are called *same side interior.*

On the other hand, the pair of angles MPB and PQD – which are non-alternate angles, one interior and one exterior – is called *corresponding*.



Refer to the diagram and name:

(a) The other pair of alternate interior angles

(b) The other pair of same side interior angles

(c) The angles that correspond to CQT and to TQD

282. In quadrilateral $ABCD$, it is given that $\vec{AB} = \vec{DC}$. What kind of quadrilateral is $ABCD$. What can be said about the vectors $\vec{AD}$ and $\vec{BC}$?

283. Given isosceles triangle *ABC* where *AB* = *BC* = 10 and the altitude from *B* has length 4. Find the length of the base. Leave your answer in simplest radical form.

284. Suppose that two of the angles of triangle *ABC* are known to be congruent to two of the angles of triangle *PQR*. What can be said about the third angles?

285. Suppose that *ABCD* is a square and that *CDP* is an equilateral triangle, with *P* outside the square. What is the size of angle *PAD*?

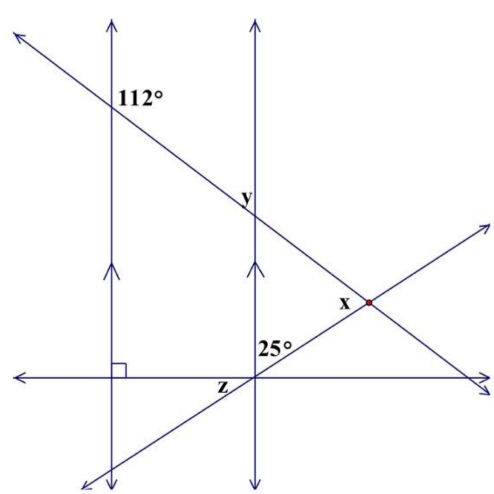
<u>GeoGebra Activity #8 – Properties of Angles formed by Parallel Lines cut by a Transversal</u>

Go to the [Avenues GeoGebra Book] and look for *Activity #8: Transversal Intersects Parallel Lines*

a. For each of the type of angles named in the table below, check the box that allows you to view that type of angle in the window.
b. Use the Measure Angle Tool  to measure a pair of those angles and record the measures in the first column.
c. Move the top slider to change the slope of the transversal. Then record the measure of another pair of the same type of angles (not the same pair that are already measured.)
d. If you see a relationship, write that down in the third column. Discuss this with your classmates if you are not sure what the relationship is.
e. Do the same for the other two types of angles named in the table below.

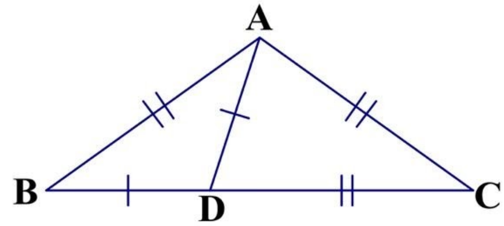
Angle Pair Type	First pair angle measures	Second pair angle measures	What is the relationship?
Corresponding			
Alternate Interior			
Same Side Interior			

286. For the diagram at the right, find the measure of the angles indicated. The arrows on the lines indicate that they are parallel. Be ready to justify your answers.



287. It is a *postulate* (statement assumed without proof) that given two parallel lines cut by a transversal, corresponding angles are congruent. Given two parallel lines cut by a transversal, how could you prove that any pair of alternate interior angles are congruent?

288. Given quadrilateral $ABCD$ with $\angle BDC \cong \angle DBA$ and $AB \cong DC$, what kind of quadrilateral is $ABCD$? Prove your conjecture.



289. In the figure at the right, it is given that BDC is straight, $BD = DA$, and $AB = AC = DC$. Find the size of angle C.

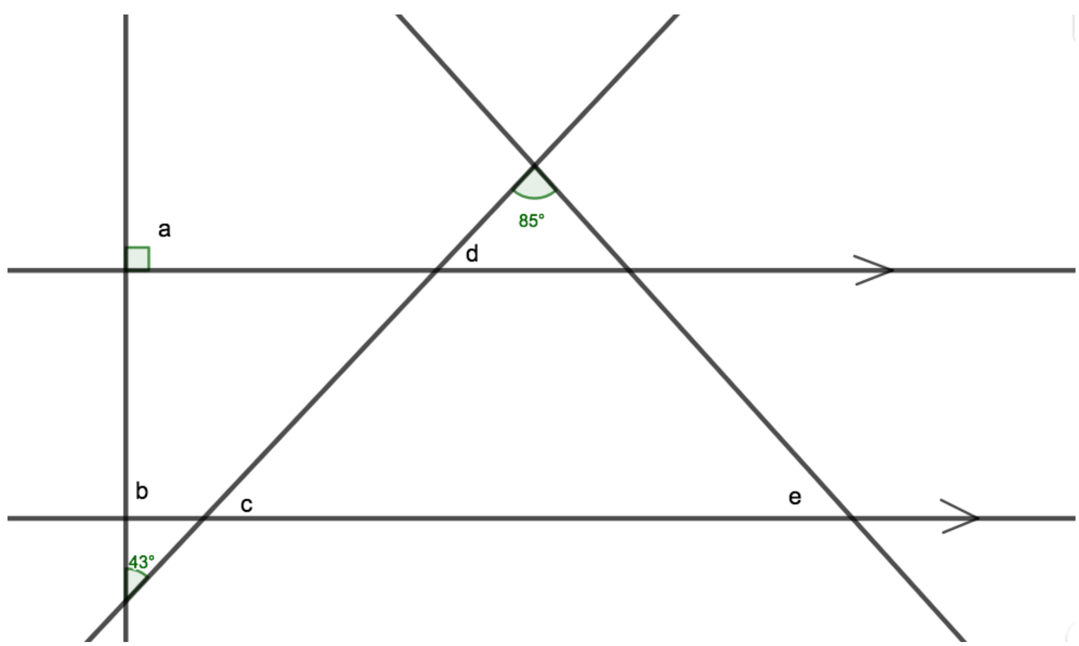
290. Read carefully and draw an accurate diagram: Triangle ABC is isosceles with AB congruent to AC. Extend segment BA to a point T (in other words, A should be between B and T). Check your diagram with classmates.

291. You know that a triangle with sides of 3, 4, and 5 is a right triangle. What would happen to the triangle if the third side changed from 5 to 4.5? From 5 to 2? From 5 to 1? Can it be less than 1?

292. If it is known that one pair of alternate interior angles is equal, what can be said about
(a) the other pair of alternate interior angles? (b) either pair of alternate exterior angles?
(c) any pair of corresponding angles? (d) either pair of non-alternate interior angles?

293. The sum of the interior angles of a triangle is the same as the measure of a straight angle. One way to confirm this is to draw a line through one of the vertices, parallel to the opposite side. This creates some alternate interior angles. Finish the demonstration.

294. In the diagram below, assume the two horizontal lines are parallel. Find the measurements of all angles labeled with a lowercase letter. Be able to justify your measurements.



295. Recall the diagram you drew of the isosceles triangle *ABC* with extended side *BA* to a point *T*. in problem 290. Use your accurate diagram to prove that angle *TAC* must be twice the size of angle *ABC*. Angle *TAC* is called one of the *exterior angles of triangle ABC*.

296. Another exercise in drawing a diagram: Draw an isosceles triangle *ABC* with *AB* = *AC*. Extend segment *AB* to a point *P* so that *BP* = *BC*. Notice that extending segment *AB* does not mean the same thing as extending segment *BA*). Compare your diagram to the person next to you. Are they identical?

297. (Continuation) Using the diagram from the previous problem, show that angle *ACP* is exactly three times the size of angle *APC*.

298. Recall that a quadrilateral that has two pairs of parallel opposite sides is called a *parallelogram.* What relationship exists between the interior angles of a parallelogram?

299. Prove the sum of interior angles in a quadrilateral is 360.

300. What would happen to a 5-12-13 triangle if the hypotenuse was shortened to 11 while the sides of 5 and 12 stayed constant? What if the hypotenuse were shortened to 7 while the remaining sides stayed the same? What about 5?

301. (Continuation) Under what conditions would a triangle with two sides of 5 and 12 be acute?

GeoGebra Activity #9– Exterior Angle of a Triangle

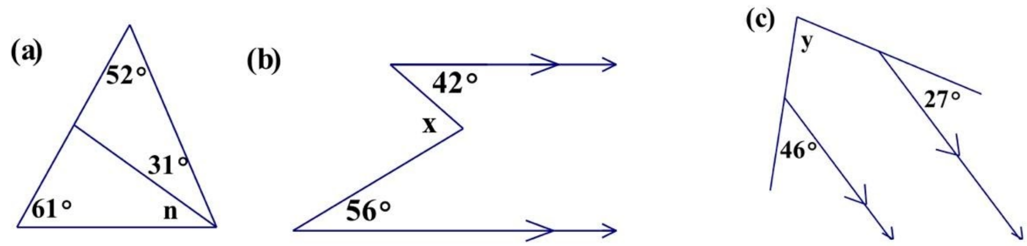
Go to the *Avenues GeoGebra Book* and look for *Activity #9: Triangle Exterior Angle.*

(a) In the applet there, you will see a diagram of the exterior angle of a triangle. What do you think the definition of an Exterior Angle of a Triangle is?
(b) The pink angle and green angle are referred to as remote interior angles with respect to this exterior angle. Where are these in relation to the exterior angle? Why do you think they are called "remote"?
(c) Use the slider and watch the animation. What can you conclude about the measure of an exterior angle of a triangle with respect to its 2 remote interior angles?
(d) Write a statement that you might be able to prove about how you might be able to find the measure of the exterior angle of a triangle. Discuss with your classmates how you might prove this statement.

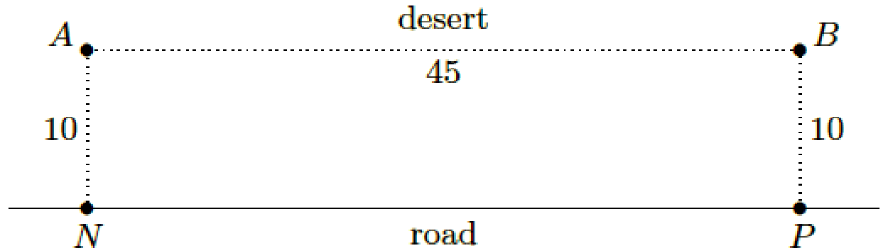
302. Triangle *ABC* has a 34-degree angle at *A*. The bisectors of angles *B* and *C* meet at point *I*. What is the size of angle *BIC*?

303. Write the Pythagorean Theorem in if... then... form. State the converse of the Pythagorean Theorem.

304. Find the size of the angles marked with letters, using the given numerical information.



305. Alex the geologist is in the desert again, 10 km from a long straight road and 45 km from base camp. The base camp is also 10 km from the road, on the same side of the road as Alex is. On the road, the jeep can do 50 kph, but in the desert sands, it can manage only 30 kph. Alex wants to return to base camp as quickly as possible. How long will the trip take?



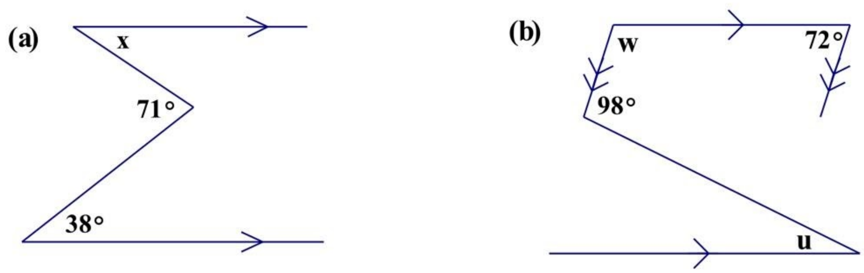
306. When you choose one vertex in a polygon and draw diagonals to every other vertex, there are *non-overlapping* triangles formed. Consider polygons with the given number of sides and complete the table:

Number of Sides of Polygon	3	4	5	6	7	8	n
Number of non-overlapping triangles	1	2					
Total Sum of the angles	180°	360°					
One Angle Measure inside a regular n-sided polygon							

307. Find an equation that says that $P = (x, y)$ is equidistant from $F = (2, 0)$ and the y-axis. Plot four points that fit this equation. The configuration of all such points P is called a *parabola*. What type of equation gives a graph that is *parabolic*?

308. Given parallelogram $PQRS$, let T be the intersection of the bisectors of angles P and Q. Without knowing the sizes of the angles of $PQRS$, calculate the size of angle PTQ. Recall that the diagonals of a parallelogram are not necessarily the angle bisectors.

309. In the figures below, find the size of the angles indicated by the letters:



310. Mark the point *P* inside square *ABCD* that makes triangle *CDP* equilateral. Calculate the size of angle *PAD*.

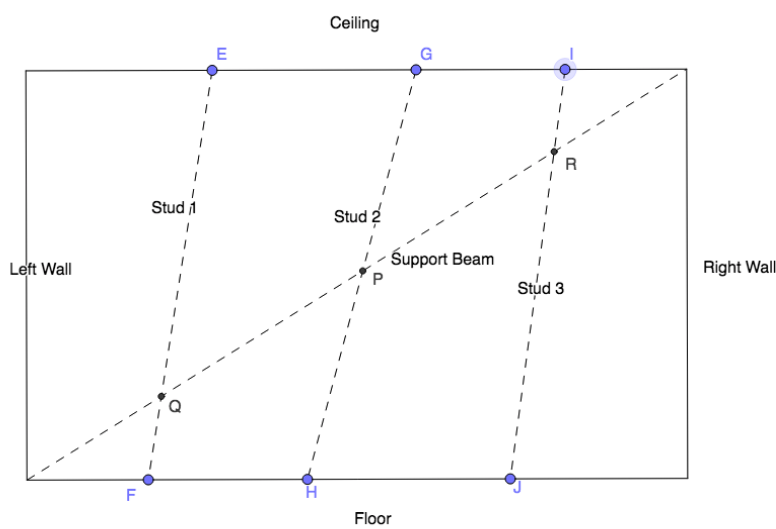
311. If a quadrilateral is a parallelogram, then both pairs of opposite angles are congruent. What is the converse of this statement? Is the converse true?

312. Consider triangle *MAC* with vertices $M = (7, -1)$, $A = (-5, 5)$ and $C = (5, -5)$.
   a. Find the circumcenter of *MAC*.
   b. How far are all of the vertices from the circumcenter?
   c. Make a conjecture about the midpoint of the hypotenuse of a right triangle. Explain.

313. Find the measure of an interior angle of a regular decagon.

314. In regular pentagon *ABCDE*, draw diagonal *AC*. What are the sizes of the angles in triangle *ABC*? Prove the segments *AC* and *DE* are parallel.

315. A carpenter's apprentice was sent to a house to line up wall studs (vertical). In doing so, the studs were supposed to be parallel to each other and the walls. However, they did not correctly nail in the studs and they shifted overnight. How could the apprentice realign the studs and be sure that they are parallel to each other again? (attribution: Anthony Dove)



316. An exercise in diagram drawing: Draw square $ABCD$ with P and Q points outside the square that make triangles CDP and BCQ equilateral. Be sure to read the description more than once before you attempt to draw the diagram.

317. The sides of an equilateral triangle are 12 cm long.
  a. How long is any altitude of this triangle?
  b. The altitude divides the triangle into two right triangles. What are the measures of the angles in these right triangles?
  c. Considering the original triangle was equilateral, how does the short side of this right triangle compare to the hypotenuse?
  d. What is the ratio of the short leg to the long leg that you found in part (a). Leave your lengths in simplest radical form.

318. If the diagonals of a quadrilateral bisect each other, then the figure is a parallelogram. Prove this statement. What about the converse? If it is true, prove it. If not, state why.

319. For your diagram from problem 316, prove whether triangle APQ is equilateral or not.

320. Draw a triangle ABC such that angles A, B, and C are acute angles. Draw the altitude from B to side AC, and label the intersection point D. Draw the line through A that is parallel to side BC. Then, Extend BD until it intersects the line drawn through A. Label K as the intersection point of BD and the line through A. Does your diagram look like your classmates?

321. Let $P = (2, 7)$, $B = (6, 11)$ and $M = (5, 2)$. Find a point D that makes $\vec{PB} = \vec{DM}$. What can you say about the quadrilateral $PBMD$?

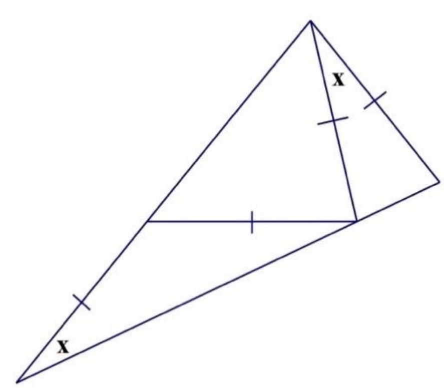
322. When translation by a vector [2, 5] is followed by translation by a vector [5, 7], the net result can be achieved by applying a single translation; what is its vector?

323. A puppy outside in a park runs along the outer boundary of a four-sided fenced-in dog park. His owner being a diligent geometry student, writes down the angles that the puppy turns at each corner. What would you guess is the sum of all four of these numbers and why?

324. Given that (1, 4) is the reflected image of (5, 2), find an equation for the *line of reflection*.

325. Recall the triangle ABC diagram that you drew in problem 320. Suppose that angle A is 59 degrees and angle B is 53 degrees. The altitude from B is still drawn to the side AC and the line through A that is parallel to side BC is also in the diagram. The altitude from B to AC has been extended until it intersects that line through A that is parallel to segment BC at the point that is called K. Now find the measure of angle AKB.

326. In the figure at the right there are two x-degree angles, and the four segments are congruent as marked. Find the value of x.



327. Jackie walks around the outer boundary of a five-sided fenced-in horse pasture writing down the degrees she turned at each corner. What is the sum of these five numbers?

328. Marty walks around the outer boundary of a seventy-sided plot of land writing down the number of degrees turned at every corner. What is the sum of these seventy numbers?|

329. The proceeding two questions illustrate the Sentry Theorem. What does this theorem say in your own words and why do you think it has been given this name?

330. Can two angle bisectors of a triangle intersect perpendicularly? Try arguing with what's called a *Proof by Contradiction*. This is when you assume the statement is true and come up with a contradiction to an already known fact.

331. A right triangle has a 24-cm hypotenuse which is twice as long as one of its legs. In simplest radical form, find the lengths of all three sides of this triangle.

332. Suppose that quadrilateral $ABCD$ has the property that AB and CD are congruent and parallel. Is this enough information to prove that $ABCD$ is a parallelogram? Explain.

333. Suppose that one of the medians of a triangle happens to be exactly half the length of the side to which it was drawn. What can be said about the angles of this triangle? Justify your response.

334. (Continuation) Prove that the midpoint of the hypotenuse of a right triangle is equidistant from all three vertices of the triangle. How does this statement relate to the preceding?

335. Kim is walking along a long straight path which happens to be the hypotenuse of a right triangle. Kim's friend Lee leaves the vertex at the right angle walking at the same rate as Kim and plans to intercept Kim's path at a point halfway along that path. They continue finishing their walk together along the hypotenuse. What is the relationship between the distances they each walked?

336. The *midsegment* of a triangle is a segment that connects the midpoints of two sides of the triangle. Given a triangle with vertices $A = (1, 7)$, $B = (5, 3)$ and $C = (-1, 1)$, find the coordinates of the midpoint of the sides of AB and AC, label the midpoints M and N, respectively. Draw the midsegment, MN

(a) Find the length of the midsegment MN and compare it to the length of BC.

(b) What can be said about the lines containing BC and MN?

337. A regular n-sided polygon has 18-degree exterior angles. Find the integer n.

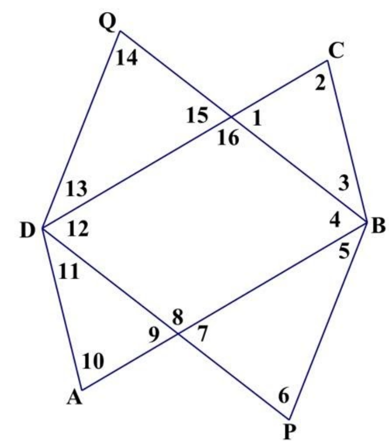
338. Alex the geologist is in the desert, 10 km from a long, straight road. Alex's jeep can go 50 kph on the road and 30 kph in the desert. Alex must return immediately to base camp which is on the same side of the road, 10 km from the road and d km from Alex. It so happens that the quickest possible trip will take the same amount of time, whether Alex uses the road or drives all the way in the desert. Find d.

339. Draw a triangle ABC, and let AM and BN be two of its medians, which intersect at G. Extend AM to the point P that makes $GM = MP$. Prove that $PBGC$ is a parallelogram.

340. (Continuation) Using the same diagram, Extend BN to the point Q that makes $GN = NQ$.

a. Find two segments in your diagram that must have the same length as BG.
b. How do the length of segments BG and GN compare?
c. What kind of quadrilateral is $PCQG$? How do you know?

341. Triangle FLB has a perimeter of 23 and $BF = \frac{1}{2}LB$. The midsegment parallel to $LB = 4$. Find the lengths of the three sides of this triangle.



342. In the figure at right, it is given that $ABCD$ and $PBQD$ are parallelograms. Which of the numbered angles must be the same size as the angle numbered 1? Give a reason for each angle.

343. Triangle PQR has a right angle at P. Let M be the midpoint of QR. Draw the altitude from P to QR and let F be the point where that altitude meets QR. Given that angle FPM is 18 degrees, find the sizes of angles Q and R.

344. Let $A = (1, 1)$, $B = (3, 5)$ and $C = (7, 2)$. Using transformations, describe how to cover a page of graph paper with non-overlapping triangles, each of which is congruent to triangle ABC.

345. (Continuation) In the pattern of lines produced by your *tessellation*, you should see triangles of many different sizes. What can you say about their sizes and shapes?

346. *Midsegment Theorem with Vectors*: Draw a triangle ABC, and let M and N be the midpoints of sides AB and AC, respectively. Express $\vec{BC}$ and $\vec{MN}$ in terms of the vectors $\vec{u}$ and $\vec{v}$ where $\vec{u} = \vec{AB}$ and $\vec{v} = \vec{AC}$.

347. Given rectangle $ABCD$, let P be the point outside $ABCD$ that makes triangle CDP equilateral, and let Q be the point outside $ABCD$ that makes triangle BCQ equilateral. Prove that triangle APQ is also equilateral.

348. Given that $ABCDEFG$…is a regular n-sided polygon, with angle $CAB = 12$ degrees, find n.

349. *Midsegment Theorem:* State the properties of the segment that connects the midpoints of two sides of a triangle.

350. Let $A = (0, 0)$, $B = (7, 2)$, $C = (3, 4)$, $D = (3, 7)$ and $E = (-1,5)$. Cameron walks the polygonal path $ABCDE$, writing down the number of degrees turned at each corner. What is the sum of these five numbers? Notice that $ABCDE$ is not a *convex* polygon.

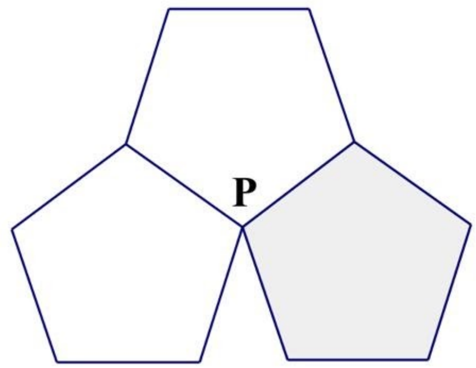
351. Using graph paper, draw a triangle by placing the vectors [6,4] and [2,8] tail-to-tail. State a vector that represents a midsegment of this triangle.

352. Is it possible for a pentagon to have interior angles 120, 120, 120, 90, and 90 degrees in this order? What about 120, 120, 90, 120, and 90 degrees? Are there other arrangements of the five angles that could have been considered? Do any of these pentagons tessellate?

353. Draw triangle *ABC* so that angles *A* and *B* are both 42 degrees. Does it seem that *AB* is longer than *BC*? Why do you think so?

354. (Continuation) Extend *CB* to *E* so that *CB* = *BE*. Mark *D* between *A* and *B* so that *DB* = *BC*, then draw the line *ED*, which intersects *AC* at *F*. Find the size of angle *CFD*.

355. The diagram at right shows three congruent regular pentagons that share a common vertex *P*. The three polygons do not quite surround *P*. Find the size of the uncovered acute angle at *P*.



356. (Continuation) If the shaded pentagon were removed, it could be replaced by a regular *n*-sided polygon that would fit exactly in the remaining space. Find the value of *n* that makes the three polygons fit perfectly.

357. You are given square *ABCD* and midpoints *M* and *N* are marked on *BC* and *CD* respectively. Draw *AM* and *BN*, which meet at *Q*. Find the size of angle *AQB*.

358. How can one tell whether a given quadrilateral is a parallelogram? In other words, how much evidence is needed to be sure of such a conclusion?

359. Draw a parallelogram *ABCD*, then attach equilateral triangles *CDP* and *BCQ* to the outside of the figure. Decide whether or not triangle *APQ* is equilateral. Explain.

360. Suppose *ABCD* is a rhombus and the bisector of angle *BDC* meets side *BC* at *F*. Prove that angle *DFC* is three times the size of angle *FDC*.

361. In triangle *ABC*, the measure of angle *A* is 100 degrees and the measure of angle *B* is 57 degrees. List the sides of the triangle in order of increasing length. Justify your answers.

362. In mathematics, a counterexample is used to show that a statement is false. Can you find a *counterexample* to show that the following statement is false? If *ab* = 5 then *a* = 1 and *b* = 5.

363. Mark *Y* inside regular pentagon *PQRST*, so that *PQY* is equilateral. Is *RYT* a straight angle? Prove your answer.

364. The midpoints of the sides of a triangle are *M* = (3, -1), *N* = (4, 3), and *P* = (0, 5). Find the coordinates of the vertices of the triangle.

365. Suppose the square *PQRS* has 15-cm sides, and that *G* and *H* are on *QR* and *PQ*, respectively, so that *PH* and *QG* are both 8 cm long. Let *T* be the point where *PG* meets *SH*. Find the size of angle *STG*, with justification.

366. (Continuation) Find the lengths of *PG* and *PT*.

367. We have discussed medians, perpendicular bisectors, altitudes, midsegments and angle bisectors of triangles.
   a. Which of these must go through a vertex of a triangle?
   b. Is it possible for a median to also be an altitude? Explain
   c. Is it possible for an altitude to also be an angle bisector? Explain.
   d. Is it possible for a midsegment to also be a median? Explain.
   e. Is it possible for a perpendicular bisector to be an altitude? Explain.

368. The diagonals of a rhombus have lengths 18 and 24. How long are the sides of the rhombus?

369. A *trapezoid* is a quadrilateral with exactly one pair of sides parallel. If the non-parallel sides have the same length, the trapezoid is *isoscele*s. Make a diagram of an isosceles trapezoid whose sides have lengths 7, 10, 19 and 10 inches. Find the *altitude* of this trapezoid (the distance that separates the parallel sides), then find the enclosed area.

370. Suppose that triangle *ABC* has a right angle at *B*, that *BF* is the altitude drawn from *B* to *AC*, and that *BN* is the median drawn from *B* to *AC*. Find angles *ANB* and *NBF*, given that angle *C* is 42 degrees.

371. If a quadrilateral is a rectangle, then its diagonals are congruent. Why must this be true? What is the converse of this statement? Is it true? If you claim it is true, attempt to prove your claim. If you believe it is not true find a counterexample.

372. The diagonals of a parallelogram always bisect each other. Is it possible for the diagonals of a trapezoid to bisect each other. You might choose to use a Proof by Contradiction for this.

373. A trapezoid has a 60 degree angle and a 45 degree angle. What are the other two angles?

374. An *n*-sided polygon has the property that the sum of the measures of its exterior angles is equal to the sum of the measures of the interior angles. Find *n*.

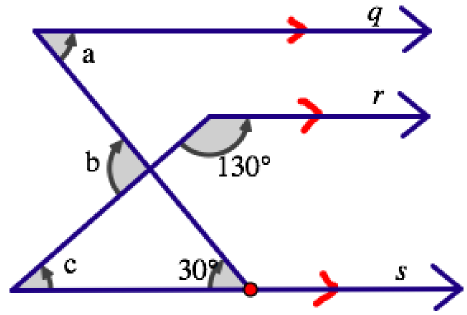
375. Trapezoid *ABCD* has parallel sides *AB* and *CD*, a right angle at *D*, and the lengths, *AB* = 15, *BC* = 10, and *CD* = 7. Find the length of *DA*.

376. A trapezoid has a 60-degree angle and a 120-degree angle. What are the other angles in the trapezoid?

377. The sides of a triangle have length 9, 12 , and 15. (This is a special triangle!)
   a. Find the lengths of the medians of the triangle.
   b. The medians intersect at the centroid of the triangle. How far is the centroid from each vertex?

378. (Continuation) Apply the same questions to an equilateral triangle of side 6.

379. In the diagram at right, the lines marked with arrows are parallel, find the angles *a*, *b*, and *c*.



GeoGebra Activity #10– The Three Parallels Theorem

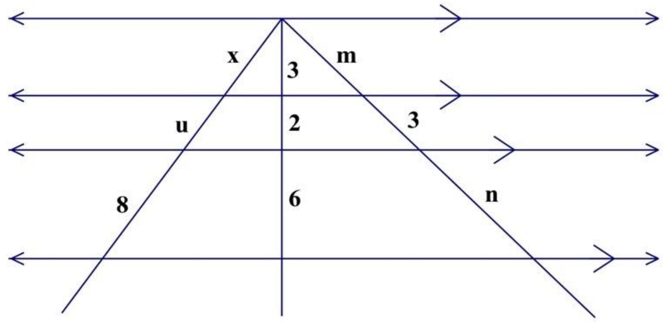
Go to the [Avenues GeoGebra Book] and look for *Activity #10: Three Parallels Theorem*

a. Interact with the following applet for a few minutes. Be sure to change the location of each BIG point. Be sure to move the slider at some point as well.
b. Talk to a classmate about what you observe changes or stays the same as you move the points or the slider. What effect is there?
c. Take the list of words in blue and attempt to write the formal statement of this theorem.

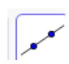
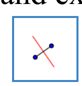
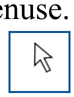
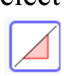
380. Find the vertices of the triangle formed by the graphs of $y = |x - 3|$ and $-x + 2y = 5$.

381. Equilateral triangles *BCP* and *DCQ* are attached to the outside of regular pentagon *ABCDE*. Is quadrilateral *BPQD* a parallelogram? Justify your answer.

382. *The Three Parallels Theorem:* If a transversal cuts three parallel lines in a given ratio, then any transversal cuts of segments of the same ratio. Use this to solve for *x*, *u*, *m* and *n* in the following diagram.

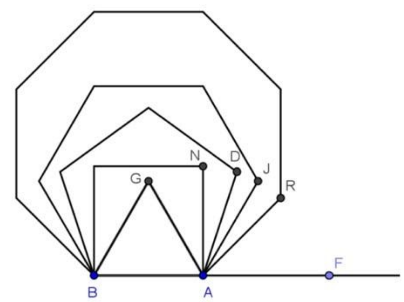


383. A line of positive slope is drawn so that it makes a 60-degree angle with the x-axis. What is the slope of this line?

384. (Continuation) In GeoGebra, with the line tool,  draw a line that crosses the x-axis and extends in the first quadrant with a positive slope. From the construction menu, construct a bisector  that connects a point on the line in the first quadrant, **B**, to the x-axis. This makes a right triangle with the first constructed line as its hypotenuse. Next, select the angle tool , click on the x-axis then the hypotenuse. Select the move tool  and move **B** so the angle is as close to 60º as possible. Now, with the slope tool , click the hypotenuse to find its slope. What is it? What exact value does this approximate? What is the ratio of the lengths of the vertical leg to the horizontal leg of the triangle? Does this ratio only work for your triangle?

385. In the diagram at the right, AGB is an equilateral triangle, AN is the side of a square. AD is the side of a regular pentagon, AJ is the side of a regular hexagon, and AR is the side of the regular octagon. AB is a side shared by all of the regular polygons. Find:

a. $\angle GAF$ b. $\angle NAR$ c. $\angle JAF$ d. $\angle GAJ$



386. What can be said about a quadrilateral if it is known that every one of its adjacent-angle pairs is supplementary?

387. If $MNPQRSTUV$ is a regular polygon, then how large is each of its interior angles? If MN and QP are extended to meet at A, then how large is angle PAN?

388. Can a triangle exist that has sides of length 23, 19 and 44? Why or why not?

389. Suppose $ABCD$ is a square with $AB = 6$. Let N be the midpoint of CD and F be the intersection of AN and BD. What is the length of AF?

390. In triangle SUN, let P be the midpoint of segment SU and Let Q be the midpoint of segment SN. Draw the line through P parallel to segment SN and the line through Q parallel to segment SU; these lines intersect at J. What appears to be true about the location of point J? Explain your answer.

391. Prove that an Isosceles trapezoid must have two pairs of equal adjacent angles.

392. (Continuation) The converse question: If a trapezoid has two pairs of equal adjacent angles, is it necessary that its two non-parallel sides have the same length? Explain.

393. Let $ABCD$ be a parallelogram with M the midpoint of DA, and diagonal AC of length 36. Let G be the intersection of MB and AC. What is the length of AG?

394. The parallel sides of trapezoid $ABCD$ are AD and BC. Given that AB, BC, and CD are each half as long as side AD, find the size of angle D.

395. Squares $COPY$ and $YAWN$ are attached to the outside of equilateral triangle NYC.

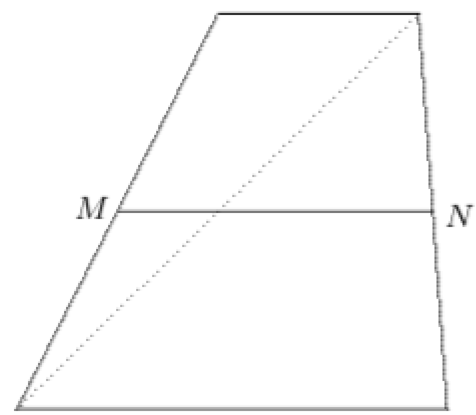
a. Draw segment PA, then find the size of angle APY.
b. Decide whether segments PN and AC have the same length, and give your reasons.

396. Use graph paper to draw non-isosceles trapezoid $ABCD$, with $A = (3, 4)$, $B = (9, 4)$, $C = (10, 1)$, and $D = (1, 1)$. Extend DA and CB until they meet at a new point E. Find the coordinates of point E, and verify that the ratios $\dfrac{AD}{DE}$ and $\dfrac{BC}{CE}$ are equal.

397. The lengths of the sides of triangle ABC are $AB = 15 = AC$ and $BC = 18$. Find the distance from A to the centroid of ABC, then find the distance from A to the circumcenter of ABC.

398. A parallelogram has two 19-inch sides and two 23-inch sides. What is the range of the possible lengths for the diagonals of the parallelogram?

399. If M and N are the midpoints of the non-parallel sides of a trapezoid, it makes sense to call the segment MN the *midline* of the trapezoid. Why? (It actually should be called the *midsegment*, of course. Strange to say, some textbooks call it the *median*). Suppose that the parallel sides of a trapezoid have lengths 7 and 15. What is the length of the midline of the trapezoid? Notice that the midline is divided into two pieces by a diagonal of the trapezoid. What are the lengths of these pieces? Does it matter which diagonal is drawn?



400. An equilateral triangle has sides of length of 12 cm. How long is the altitude of this triangle?

401. Dana buys a piece of carpet that measures 20 square yards. Will Dana be able to completely cover a rectangular floor that measures 12 feet and 4 inches by 16 feet and 8 inches?

402. Interpret each of the following shapes as a trapezoid with two parallel bases of given length, and find the length of each midline.
   a. Top base of length 0 and bottom base of length 10.
   b. Top base of length 10 and bottom base of length 10.
   c. Top base of length 6 and bottom base of length 10. From the previous two examples, how might you conjecture to find the length of the midline? Try to justify your answer.

403. Draw the lines $y = 0$, $y = \frac{1}{2}x$, and $y = 4x$. Use GeoGebra to measure the angle that the line $y = \frac{1}{2}x$ makes with the x-axis. Using your intuition, make a guess at what the angle is that the line $y = 4x$ makes with the x-axis. Now measure it. Explain if your intuition leads to the correct measurement. Create a table with some slopes of lines and measures of angles so that you can compare. Is there a relationship that you observe between the slope of the line and the size of the angle that it makes with the x-axis?

404. Recall that the midline of a trapezoid is the segment that joins the midpoints of the non-parallel sides. Prove that the midline of a trapezoid splits the trapezoid into two new trapezoids.

405. The altitudes of an equilateral triangle measure 12 cm. How long are the sides?

406. It is given that the sides of an isosceles trapezoid have lengths 3, 15, 21 and 15 cm. Make a diagram. Show that the diagonals intersect perpendicularly.

407. Given a triangle, you know the following result: *The points where two medians intersect (the centroid) is twice as far from one end of a median as it is from the other end of the same median.* Improve the statement so that the reader knows which end of the median is which.

408. The parallel bases of a trapezoid have lengths 12 and 18 cm. Find the lengths of the two segments into which the midline of the trapezoid is divided by one of the diagonals.

409. The sides of a square have length 10. How long are the diagonals of the square? Keep your answer in simplest radical form. What would your answer have been if the side had been 6?

410. In triangle *ABC*, let *M* be the midpoint of *AB* and *N* be the midpoint of *AC*. Suppose that you measure *MN* and find it to be 7.3 cm long. How long would *BC* be if you measured it? What should be true about angles *AMN* and *ABC*?

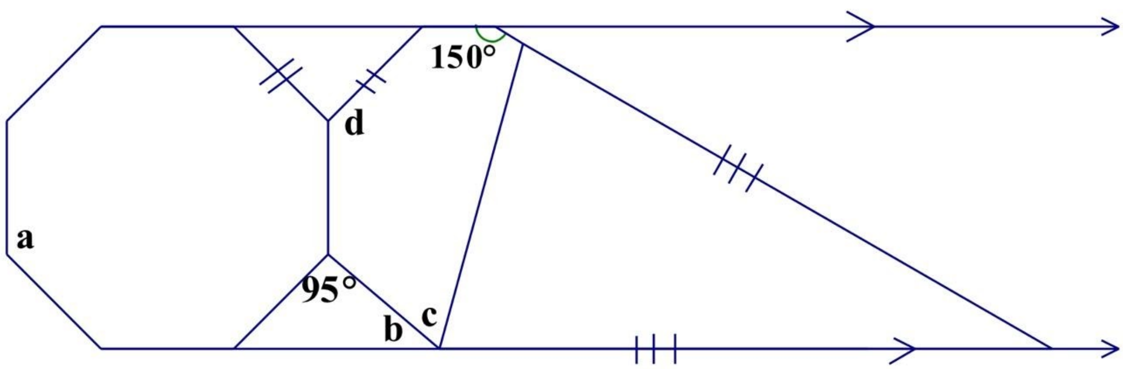
<u>GeoGebra Activity #11 – The Angle Bisector Theorem</u>

Go to the *Avenues GeoGebra Book* and look for *Activity #11: Angle Bisector Proportionality Theorem.*

1. Before you click either of the checkboxes, select a vertex of the triangle and move it around. Is there a special location that the angle bisector always stays in? What do you think?
2. Click the checkbox that says "Click here first" and make note of the lengths that you see. Move the triangle around and see if you notice anything special about the lengths. You might want to try making the triangle into some special triangles – right, isosceles, equilateral, obtuse, etc. Do you notice any patterns in the lengths of the segments that are shown?
3. Now Click the checkbox that says "Click here second" and see if your conjectures were right. Move the triangle around some more and check if it is always true.
4. Write a statement that might be considered "The Angle Bisector Theorem."

411. A bell rope, passing through the ceiling above, just barely reaches the bell tower floor. When the bell ringer pulls the rope to the wall, keeping the rope taut, it reaches a point that is three inches above the floor. The distance from the wall to the rope when the rope is hanging freely is four feet. How high is the ceiling? It is advisable to draw a clear diagram for this problem.

412. In the diagram below the octagon is regular. Find the measures of the angles labeled, a, b, c and d.



413. Mark *A*=(0, 0) and *B*=(10, 0) on graph paper or on GeoGebra and use the "Angle of Given Size" tool  to construct the line of positive slope through *A* that makes a 25-degree angle with *AB*. Calculate (approximately) the slope of this line making suitable measurements.

414. (Continuation) Go to https://www.desmos.com/scientific and make sure it says you are in degree mode 

. Press the TAN button to enter the expression TAN(25), then press return or the blue arrow button. What do you notice? Make a conjecture about what the TAN function does.

415. When the sun has risen 32 degrees above the horizon, a tenth grader casts a shadow that is 9 feet 2 inches long. How tall is this student, to the nearest inch?

416. The diagonals of a non-isosceles trapezoid divide the midline into three segments whose lengths are 8 cm, 3 cm and 8 cm. How long are the parallel sides? From this information is it possible to infer anything about the distance that separates the parallel sides? Explain.

417. A line drawn parallel to the side BC of triangle ABC intersects side AB at P and side AC at Q. The measurements $AP = 3.8$ cm, $PB = 7.6$ cm and $AQ = 5.6$ cm are made. If segment QC were now measured, how long would it be?

418. Given $A = (0, 6)$, $B = (-8,0)$, and $C = (8,0)$, find the coordinates for the circumcenter of triangle ABC.

419. Rearrange the letters in the word *doctrine* to spell a familiar mathematical word.

420. How does the value of the tangent of an angle change as an angle increases from 0 to 90 degrees? Is there a direct relationship between the slope and the angle measure? Justify your answer.

421. Standing 50 meters from the base of a fir tree, Rory measured an angle of elevation of 33 degrees to the top of the tree with a protractor. The *angle of elevation* is the angle formed by the horizontal ground and an ant's line-of-sight ray to the top of the tree. How tall was the tree?

422. Given regular hexagon $BAGELS$, show that SEA is an equilateral triangle.

423. You are creating a tent for a sleepover in your backyard. You have a 36 ft. long tarp that you are going to use and hang over a clothes line symmetrically to form the tent. Looking at it from the side with a cross-section that looks like an isosceles triangle, what are the possible widths of the base of the tent? What realistic assumptions are you making?

http://www.rockabyebabymusic.com/blog/tag/tent/

424. Standing on a cliff 380 meters above the sea, a lookout soldier sees an approaching ship and measures its *angle of depression*, obtaining 9 degrees. How far from shore is the ship?

425. (Continuation) Now the lookout sights a second ship beyond the first. The angle of depression of the second ship is 5 degrees. How far apart are the ships?

426. Hexagon $ABCDEF$ is regular. Prove that segments AE and ED are perpendicular.

427. Find the tangent of a set of stairs in your home or school. How could we use that tangent to tell whose stairs are the steepest in the class?

428. Find the coordinates for the point P where the line $y = x$ intersects the line $2x + 3y = 24$. Then calculate the distances from P to the axis intercepts of $2x + 3y = 24$. The Angle-Bisector Theorem makes a prediction about these distances – what is the prediction?

429. Alex and Skyler are in the park playing angry birds on the iPads one day and spy a bird in a tree that they know is 25 feet away from them. Alex decides to measure the angle of elevation to the top of the tree and sees that it is 60 degrees. Skyler says, "I wonder how tall that tree is?" Alex replies, "Well, it can't be more than 50 feet tall, that's for sure." How does Alex know this? (without using a calculator!)

430. What is the relationship between the length of the hypotenuse and the length of the legs in a 45-45-90 triangle?

431. What is the radius of the smallest circle that encloses an equilateral triangle with 12-inch sides? What is the radius of the largest circle that will fit inside the same triangle?

432. Suppose that *ASCENT* is a regular hexagon, and that *ARMS*, *BATH*, *LINT*, *FEND*, *COVE* and *CUPS* are squares attached to the outside of the hexagon. Decide whether dodecagon *LIDFVOUPMRBH* is regular and give you reasons.

433. Let $A = (0, 0)$, $B = (4, 0)$, and $C = (4, 3)$. Measure angle *CAB* with a protractor (you must have a protractor for this). What is the slope of *AC*? Use your calculator to compare the tangent of the angle you measured with the slope. By trial-and-error, find an angle that is a better approximation of the measure of angle *CAB*.

434. (Continuation) At https://www.desmos.com/scientific , press the func button and then press the TAN^{-1} button. Enter $\text{TAN}^{-1}(.75)$. Compare this answer with the approximation you obtained for the measure of angle *CAB*. What does the TAN^{-1} button do? (TAN^{-1} is said as "inverse tangent," it is not an exponent).

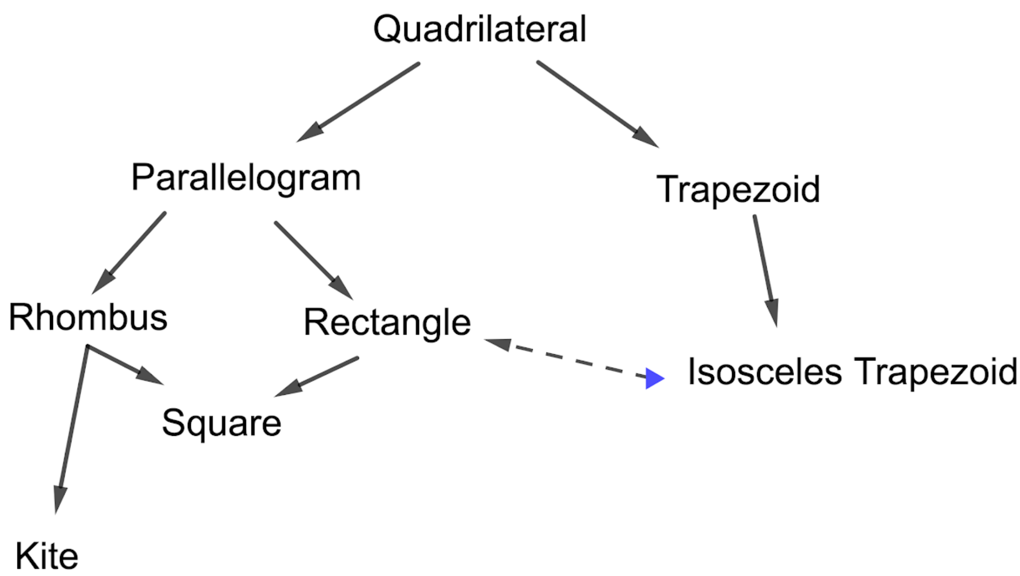
435. An isosceles trapezoid has sides of lengths 9, 10, 21 and 10. Find the distance that separates the parallel sides, then find the length of the diagonals. Find the angles of the trapezoid, to the nearest tenth of a degree.

436. Suppose that *ANGEL* is a regular pentagon and that *CANT, HALF*, *ROLE*, *KEGS* and *PING* are squares attached to the outside of the pentagon. Show that decagon *PITCHFORKS* is equiangular. Is this decagon equilateral?

437. A five foot Avenues student casts an eight-foot shadow. How high is the sun in the sky? This question is not asking for a distance, by the way.

438. Inside regular pentagon *BRIAN* is marked point *U* so that *BRU* is equilateral. Decide whether or not quadrilateral *AIUN* is a parallelogram and give your reasons.

Justify the following Tree Diagram and place a check in the table of the properties that each type of quadrilateral holds.



Property	Parallelogram	Rectangle	Rhombus	Square	Kite	Trapezoid	Isosceles Trapezoid
Opposite sides are parallel							
Opposite sides are congruent							
Only one pair of opposite sides is parallel to each other							
Opposite pairs of angles are congruent							
Consecutive angles are supplementary							
Diagonals bisect each other							
Diagonals are congruent							
Diagonals perpendicularly bisect each other							
Diagonals are perpendicular							
Diagonals bisect opposite angles							
Two pairs of angles are congruent							
Only one diagonal is the perpendicular bisector of the other							

439. One day at the beach, Kelly flies a kite, whose string makes a 37-degree elevation angle with the ground. Kelly is 130 feet from the point directly below the kite. How high above the ground is the kite, to the nearest foot? What (unrealistic) assumptions did you make in answering this question?

440. Suppose that *PQRS* is a rhombus, with *PQ* = 12 cm and a 60-degree angle at *Q*. How long are the diagonals *PR* and *QS*?

441. A triangle whose sides are 6, 8, and 10, and a circle, whose radius is *r*, are drawn so that no part of the triangles lies outside the circle. How small can *r* be?

442. Diagonals *AC* and *BD* of regular pentagon *ABCDE* intersect at *H*. Decide whether or not *AHDE* is a rhombus, and give your reasons.

443. Draw a regular pentagon and all five of its diagonals. How many isosceles triangles can you find in the picture? How many *scalene triangles* can you find?

444. Let *A* = (3, 1), *B* = (9, 5), and *C* = (4, 6). Your protractor should tell you that angle *CAB* is about 45 degrees. Explain why angle *CAB* is in fact exactly 45 degrees.

445. A triangle has sides in a ratio 1:2: $\sqrt{3}$. Draw a triangle with this scale with specific lengths. What can you say about this triangle?

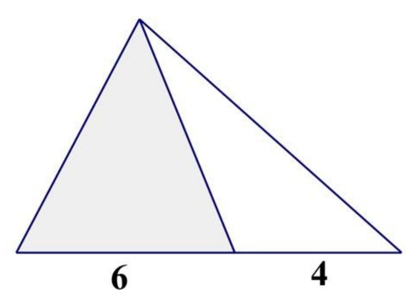
446. Let *ABCD* be a square. Mark midpoints *M*, *N*, *O*, *P* on *AB*, *BC*, *CD*, and *DA*, respectively. Draw *AN*, *BO*, *CP*, and *DM*. Let *Q* and *R* be the intersections of *AN* with *DM* and *BO*, respectively, and let *S* and *T* be the intersections of *CP* with *BO* and *DM*, respectively. Prove as much as you can about this figure, especially quadrilateral *QRST*.

447. In an Avenues ninth grade class there are about 90 students and the ratio of students who take Mandarin as a second language to students who take Spanish is 4:5.
   a. How many students in this class are taking Spanish?
   b. How many students would you expect who take Mandarin to be in a Math 2 class that has 20 students?

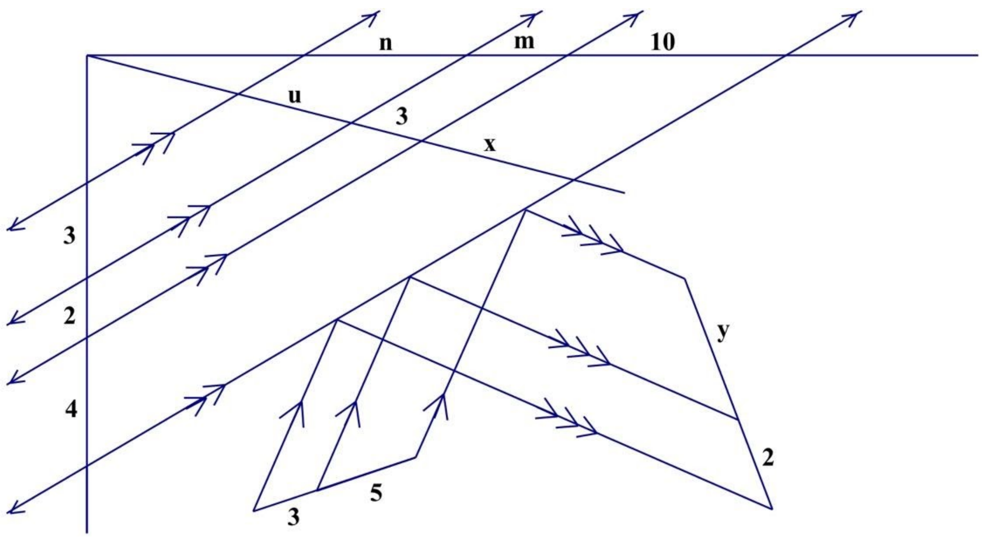
448. Find the equation of a line passing through the origin that makes an angle of 52 degrees with the *x*-axis.

449. What are all of the special right triangles? In geometry it is very useful to know many right triangles that have special properties. Which right triangles so far this year have been helpful in our work? Explain as many as you think of and why they have been "special" in their own way. Be as specific as possible.

450. In the figure at right, the shaded triangle has area 15. Find the area of the unshaded triangle.



451. In the figure below, find the lengths of the segments indicated by the letters. Parallel lines are indicated by the arrows.



452. To the nearest degree, how large are the congruent angles of an isosceles triangle that is exactly as tall as it is wide?

453. Rectangle *ABCD* has *AB*=16 and *BC*=6. Let *M* be the midpoint of side *AD* and *N* be the midpoint of side *CD*. Segments C*M* and *AN* intersect at *G*. Find the length of *AG*.

454. An estate of $362880 is to be divided among three heirs, Alden, Blair and Cary. According to the will, Alden is to get two parts, Blair three parts and Cary four parts. How much money in dollars and cents does each heir receive?

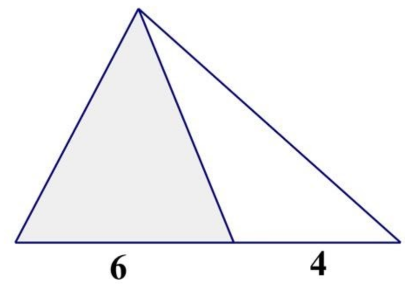
455. The area of a parallelogram can be found by multiplying the distance between two parallel sides (the height, altitude) by the length of either of those sides. Explain why this formula works. Draw a picture and explain why the phrase "two parallel sides" was used in this problem instead of "base."

456. Given a rectangular card that is 5 inches long and 3 inches wide, what does it mean for another rectangular card to have the *same shape*? Describe a couple of examples.

457. The perimeter of a square is 36. What is the length of a diagonal of the square? What is the length of the diagonal if the area of the square were 36?

458. Using GeoGebra, plot the points *A* = (0, 0), *B* = (4, -3), *C* = (6, 3), *P* = (-2, 7), *Q* = (9, 5), and *R* = (7, 19). Measure the angles of triangles *ABC* and *PQR*. Create ratios of the lengths of the corresponding sides. Find justification for any conclusions you make. (If you choose not to use technology, leave all answers in simplest radical form).

In the following list of true statements, find **(a)** the four pairs of statements whose converses are also in the list; **(b)** the statement that is a definition; **(c)** the statement whose converse is false; **(d)** the Sentry Theorem; **(e)** the Midsegment Theorem; **(f)** The Three Parallels Theorem; **(g)** The Centroid Theorem. Note: Not all statements are used.



A. If a quadrilateral has two pairs of parallel sides, then its diagonals bisect each other.

B. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral must be a parallelogram.

C. If a quadrilateral is equilateral, then it is a rhombus.

D. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

E. If a quadrilateral has two pairs of equal adjacent sides, then its diagonals are perpendicular.

F. If one of the medians of a triangle is half the length of the side to which it is drawn, then the triangle is a right triangle.

G. If a segment joins two of the midpoints of the sides of a triangle, then it is parallel to the third side and is half the length of the third side.

H. Both pairs of opposite sides of a parallelogram are congruent.

I. The sum of the exterior angles of any polygon – one at each vertex – is 360.

J. The median drawn to the hypotenuse of a right triangle is half the length of the hypotenuse.

K. If two lines are intersected by a transversal so that alternate interior angles are equal, then the lines must be parallel.

L. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is in fact a parallelogram.

M. If one pair of opposite sides of a quadrilateral are both parallel and equal in length then the quadrilateral is a parallelogram.

N. If three parallel lines intercept equal segments on one transversal, then they intercept equal segments on every transversal.

O. Both pairs of opposite angles of a parallelogram are congruent.

P. The medians of any triangle are concurrent at a point that is two thirds of the way from any vertex to the midpoint of the opposite side.

Q. An exterior angle of a triangle is the sum of the two nonadjacent interior angles.

459. In trapezoid $ABCD$, AB is parallel to CD and AB =10, BC = 9, CD = 22, and DA = 15. Points P and Q are marked on BC so that $BP = PQ = QC = 3$, and points R and S are marked on DA so that $DR = RS = SA = 5$. Find the lengths PS and QR.

460. One figure is *similar* to another figure if the vertices of the first figure can be matched with the vertices of the second figure in such a way that corresponding distances are proportional. In other words, there is a *ratio of similarity*, k, such that every distance on the second figure is k times the corresponding distances on the first figure.

a. In GeoGebra plot the points $K = (1, -3)$, $L =(4, 1)$, $M = (2, 3)$, $P = (6, 5)$, $Q = (2, 5)$ and $R = (7, -2)$.

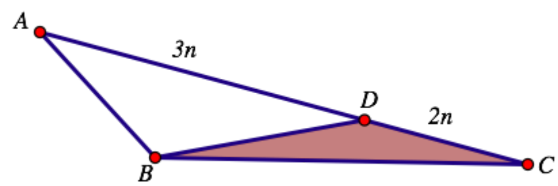
b. Is triangle KLM similar to triangle RPQ? Justify with measurement from GeoGebra. What is the ratio of similarity?

c. Would it be correct to say that triangle MKL is similar to triangle RQP?

461. Draw a right triangle ABC, with the right angle labeled B and the altitude drawn to the hypotenuse labeled BK. Give an argument as to why all three triangles, AKB, ABC and BKC would have the same angle measurements. Be as precise as possible.

462. *The Varignon quadrilateral*: A quadrilateral has diagonals of lengths 8 and 10. The midpoints of the sides of this figure are joined to form a new quadrilateral. What is the perimeter of the new quadrilateral? What is special about it?

463. The figure shows a triangle whose one side AC divided by point D into two parts as shown. If the shaded area is 24 square units, find the area of the unshaded triangle.



464. One base of a trapezoid has length 36 cm. If the length of the midline is 20, find the lengths of the segments that the two diagonals cut the midline into.

465. A rectangle $ABCD$ has dimensions AB = 5 and BC = 12. Let M be the midpoint of BC and let G be the intersection of AM and diagonal BD. Find BG and AG.

466. You are going to construct a right triangle with a 27-degree angle.

a. In GeoGebra, draw a segment that is 15 cm long. This will serve as the hypotenuse of a right triangle.

b. Use the "Angle-with-Given-Size" tool to create an angle that measures 27 degrees clockwise from one endpoint of the segment.

c. Construct a ray from that vertex to the new point that measures 27 degrees. This is a leg.

d. With the segment drawing tool, draw the other leg of the right triangle forming approximately 90 degrees.

e. On GeoGebra, read the length of the side that is opposite the 27-degree angle. What is the ratio of that length to 15?

f. Compare your answer with the value you obtain from a calculator (like desmos.com/scientific) when you enter *sin*(27) in degree mode.

467. (Continuation) Repeat the construction process on GeoGebra by creating a right triangle that has a 10-cm hypotenuse and a 65 degree angle. Write a definition of the sine function.

468. What is the length of the altitude of an equilateral triangle with perimeter 36?

469. In triangle *ABC*, points *M* and N are marked on sides *AB* and *AC*, respectively so that *AM:AB* = 1:3 = *AN:AC*. Why are segments *MN* and *BC* parallel?

470. What are the angle sizes in a trapezoid whose sides have lengths 6, 20, 6, and 26?

471. There are many ways to justify the area formula for a trapezoid which is $A = \frac{1}{2}(b_1 + b_2) \times h$. Find one way to justify this equation by splitting the trapezoid into two triangles. Find at least one other way.

472. To actually draw a right triangle that has a 1-degree angle and measure its sides accurately is difficult. To get the sine ratio for a 1-degree angle, however, there is an easy way – just use your calculator. Is the ratio a small or a large number? Explain why. How large can a sine ratio be?

473. If two sides of a triangle are 5 and 10, what is the range of values for the third side? How do you know?

474. What is the size of the acute angle formed by the x-axis and the line $3x + 2y = 12$?

475. Alden, a passenger on a boat moved 15 miles due north of a straight, east-west coastline, has become ill and has to be taken ashore in a small motorboat, which will meet an ambulance at some point on the shore. The ambulance will then take Alden to the hospital, which is 60 miles east of the shore point closest to the boat. The motorboat can travel at 20 mph and the ambulance at 90 mph. In what direction should the motorboat head to minimize the travel time to the hospital? Answer with an angle.

476. *AA Similarity Postulate*: If two corresponding angles of a triangle are equal in size to the angles of another triangle, then the two triangles are similar. Why does this statement seem to be true? What is the converse? Is it true?

477. If triangle *ABC* has a right angle at *C*, the ratio *AC:AB* is called the *sine ratio* of angle *B,* or simply the sine of B, and is usually written sin(B). What should the ratio *BC:AB* be called? Without using your calculator, can you predict what the value of the sine ratio for a 30-degree angle is? How about the sin ratio for a 60-degree angle?

478. Using GeoGebra, graph the triangle *PQR* whose vertices are *P* = (3, -1), *Q*=(1, 2), and *R* = (4, 3). Compare the sides and angles of the image triangle *P'Q'R'* where *P'*= (9, -3), *Q'* = (3, 6), and *R'* = (12, 9) with the corresponding parts of PQR. This transformation is an example of a *dilation.* What would you say is true about a triangle that is dilated?

479. In triangle *ABC,* it is given that *AB* = 4, *AC* = 6, and *BC* = 5. The bisector of angle *BAC* meets *BC* at *D*. Find the lengths *BD* and *CD*.

480. Let A = (1, 2), B = (8, 2) and C = (7, 10). Find an equation for the line that bisects angle *BAC*.

481. A regular n-sided polygon has exterior angles of m degrees each. Express m in terms of n. For how many of these regular examples is m a whole number.

GeoGebra Activity #12 – Introduction to Dilations

Go to the [Avenues GeoGebra Book]() and look for *Activity #12: Introduction to Dilations*

a. What happens when you move the center of dilation?
b. What happens when the scale factor is changed? Be specific.
c. With the slope tool, measure the slope of the lines AC, A'C', AB, A'B', and BC, B'C'. What do you notice? Why do you think this happens?
d. How would you best describe the location of the center of dilation? With the selection tool move the original vertices of the triangle ABC. Describe what happens. Does this affect the location of point O?

482. Out for a walk in Chicago, Morgan measures the angle of elevation to the distant Willis Tower, and gets 3.6 degrees. After walking one km directly toward the building, Morgan finds that the angle of elevation has increased to 4.2 degrees. Use this information to calculate the height of the Willis Tower and how far Morgan is from it now.

483. How tall is an isosceles triangle with a base that is 30 cm long and base angles that are 72 degrees?

484. Compare the quadrilateral whose vertices are $A = (0, 0)$, $B = (6, 2)$, $C = (5, 5)$, $D = (-1, 3)$ with the quadrilateral whose vertices are $P = (9, 0)$, $Q = (9,2)$, $R = (8, 2)$, and $S = (8, 0)$. Show that these two figures are similar. Is there a dilation that takes $ABCD$ to $PQRS$?

485. Write an equation using the distance formula that says that a point, $P = (x, y)$ is 5 units from the origin. Plot several such points. What is the configuration of all such points called? How many are lattice points?

486. (Continuation) Consider the distance, $d = 5$, as a hypotenuse, explain how you could use the Pythagorean Theorem to obtain the same equation.

487. Does a dilation transform any figure into a similar figure? If you know that two triangles are similar, does that mean that they are dilations of one another?

488. What is the length of a side of an equilateral triangle whose altitude is 16? How do you describe the length of the side in terms of the altitude?

489. When you take the $sin(30)$ using your calculator you get 0.5. What do you think $sin^{-1}(0.5)$ is? Use your calculator to test your conjecture. Find $sin^{-1}(0.3)$ and $sin^{-1}(3/5)$. What do these values represent?

490. When triangle ABC is similar to triangle PQR with A, B and C, corresponding to P, Q and R, respectively, it is customary to write $\Delta ABC \sim \Delta PQR$. Suppose that $AB = 4$, $BC = 5$, $CA = 6$, and $RP = 9$. Find PQ and QR.

491. In northwest Massachusetts, Bailey is driving along Route 2 from Williamstown to Shelburne Falls and sees a sign that says “7% Grade” along the very mountainous highway. The grade of a road is the slope at which it is declining. After driving for 2000 feet Bailey comes to a stop sign. How much altitude was lost?

492. (Continuation) How far would someone have to drive to lose one mile of altitude on the same road?

493. The floor plan of a house is drawn with a ratio of 1/8 inch = 1 foot. On the plan, the kitchen measures 2 inches by 2 ¼ inches. What are the dimensions of the kitchen in the house?

494. One triangle has sides that are 5 cm, 7 cm and 8 cm long; the longest side of a similar triangle is 6 cm long. How long are the other two sides?

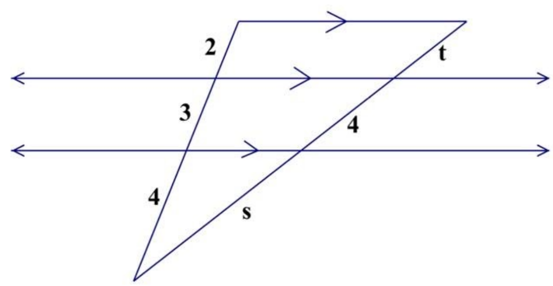
495. *Right Triangle Similarity*: Draw an arbitrary right triangle. Why does the altitude drawn from the right angle vertex to the hypotenuse create three right triangles? Try to justify your answer in a few sentences and a diagram.

496. You are given a right triangle where the altitude drawn to the hypotenuse splits the hypotenuse into two segments with length 3 and 9. How long is the altitude?

497. To the nearest tenth of degree, find the sizes of the acute angles in a 5-12-13 triangle and in a 9-12-15 triangle. Using this information, find the angles in a 13-14-15 triangle (Note: is this latter triangle a right triangle?)

498. Atiba wants to measure the width of the Hudson River. Standing at Chelsea Pier, point *C*, Atiba sights a pier, *P*, at the nearest point across the river. Atiba then walks south to the 11th Ave. pier which is 50 meters away from *C* at a point *R* and makes *PRC* a right angle. Atiba measures the angle *RPC* to be 76.8 degrees. How wide is the river?

499. In the diagram at the right, find t and s. What method did you use? Try to find another method.



500. Is it possible to draw a triangle with the given sides? If it is possible, state whether it is acute, obtuse or right. If it is not possible, say no and sketch why.

a. 9 , 6, 5
b. $3\sqrt{3}$, 9, $6\sqrt{3}$
c. 8.6, 2.4, 6.2

501. Alex the geologist is in the desert, 18 km from a long, straight road and 72 km from base camp, which is also 18 km from the road, on the same side of the road Alex is. On the road, the jeep can do 60 kph, but in the desert sands, it can manage only 32 kph.

a. Describe the path that Alex should follow to return to base camp most quickly.
b. If the jeep were capable of 40 kph in the desert, how would your answer be affected?

502. Triangle *ABC* has *AB* = 12 = *AC* and angle *A* is 120 degrees. Let *F* and *D* be the midpoints of sides *AC* and *BC*, respectively, and *G* be the intersection of segments *AD* and *BF*. Find the lengths of *FD*, *AD*, *AG*, *BG* and *BF*.

503. The parallel sides of a trapezoid have lengths *m* and *n*. The diagonals of the trapezoid divide the midline into three pieces. In terms of *m* and *n,* how long are the pieces?

504. A triangle has a 60-degree angle and a 45-degree angle and the side opposite the 45-degree angle is 240 mm long. To the nearest mm, how long is the side opposite the 60-degree angle?

505. The area of an equilateral triangle with *m*-inch sides is 8 square inches. What is the area of a regular hexagon that has *m*-inch sides (Hint: you do not have to find the length of *m*).

506. Rory, Leo and Jo win $429,184 in a lottery. They decide to divide the winnings so that Rory and Leo get the same amount and Jo twice as much as each of them. Exactly how much money does each person receive?

507. In a triangle whose sides have lengths 3, 4 and 5

a. How long is the bisector of the larger of the two acute angles?
b. How long is the bisector of the right angle?

508. To the nearest tenth of a degree, find the sizes of the acute angles in the 7-24-25 right triangle and in the 8-15-17 right triangle. This information then allows you to calculate the sizes of all angles in the 25-51-52 triangle. Show how to do it.

509. The parallel sides of a trapezoid have length 9 and 12. Draw one diagonal, dividing the trapezoid into two triangles. What is the ratio of their areas? If the other diagonal had been drawn instead, would this have affected your answer?

510. A parallelogram has 10-inch and 18-inch sides and an area of 144 square inches.

a. How far apart are the 18-inch sides?
b. How far apart are the 10-inch sides?
c. What are the angles of the parallelogram?
d. How long are its diagonals?

511. Write an equation that describes all of the points on the circle whose center is at the origin and whose radius is (a) 13; (b) 6; (c). $\sqrt{5}$; (d) *r*

512. If the lengths of the midsegments of a triangle are 6, 9 and 11, how long is the triangle's perimeter?

513. Draw a right triangle that has an 18-cm hypotenuse and a 70-degree angle. To within 0.1 cm, measure the leg adjacent to the 70-degree angle and express your answer as a fraction of the hypotenuse. Compare your answer with the value obtained from a calculator when you enter *cos*(70) in degree mode. This is an example of the *cosine ratio*.

514. Consider the triangles defined by $K = (1, -3)$, $L = (4, 1)$, and $M = (2, 3)$ and $P = (6, 5)$, $Q = (2, 5)$ and $R = (7, -2)$. Plot these points and determine whether triangle *KLM* and *PQR* are similar or dilations of each other. Give specific information such as the ratio of similarity (if there is one).

515. A rhombus has four 6-inch sides and two 120 degree angles. From one of the vertices of the obtuse angles, the two altitudes are drawn, dividing the rhombus into three pieces. Find the areas of the three pieces.

516. When moving a lot of plates during a renovation, a restaurant owner packed those round plates in square boxes with perimeter 36 inches. If the plates fit snugly in one stack inside the box, one plate per layer, what is the circumference of a single plate?

517. (*Continuation*) Each of the restaurant's saucers has a circumference of 12.57 inches. Can four saucers fit on a single layer in the same square box? Justify your answer.

518. Graph the circle whose equation is $x^2 + y^2 = 64$. What is its radius? How do you know? What do the equations $x^2 + y^2 = 1$, $x^2 + y^2 = 40$, and $x^2 + y^2 = k$ all have in common? How do they differ?

519. Let $A = (0, 5)$, $B = (-2, 1)$, $C = (6, -1)$, and $P = (12, 9)$. Let A', B' and C' be the midpoints of segments *PA*, *PB* and *PC*, respectively. After you make a diagram, identify the magnitude of dilation that transforms triangle *ABC* onto $A'B'C'$. You may use GeoGebra to answer this question.

520. To the nearest tenth of a degree, find the sizes of the acute angles in the right triangle whose long leg is 2.5 times as long as its short leg.

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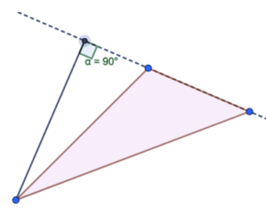
521. Pat has \$48 and Kim has \$24, so the ratio of Pat's money to Kim's money is 2:1. If they both spent \$5, is the ratio still 2:1? Explain how they could spend their money so that the ratio of Pat's money to Kim's money remains 2:1.

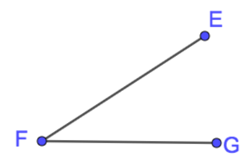
AAA Similarity Two triangles are sure to be similar if all angles are equal in size.

Adjacent Two vertices of a polygon that are connected by an edge. Two edges of a polygon that intersect at a vertex. Two angles of a polygon that have a common side.

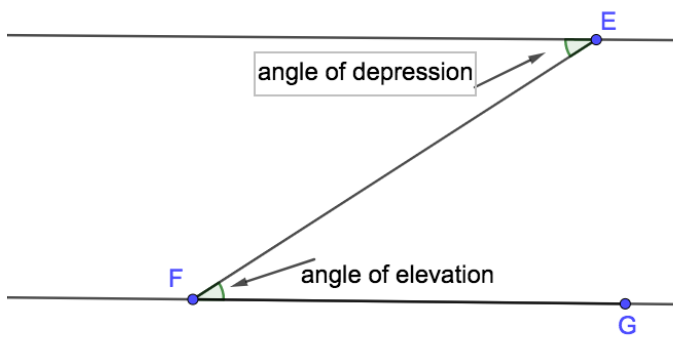
Altitude In a triangle, an altitude is a segment that joins one of the three vertices to a point on the opposite side, the intersection being perpendicular. In obtuse triangles, it may be necessary to extend a side of the triangle in order to meet an altitude. The length of this segment is also called the altitude, as is the distance that separates the parallel sides of a trapezoid.



Angles Can often be identified by a single letter, but sometimes three letters are necessary. The angles shown can be *F, EFG* or *GFE*.



Angle of Depression (or Elevation) Angle formed by a horizontal ray and a line-of-sight ray that is below the horizontal (depression) or above the horizontal (elevation). See diagram below.



AAS Criterion One of the *triangle congruence criteria*, which states that when two corresponding angles and a side that is not in between those angles have the same sizes as the corresponding angles and side of another triangle the two triangles must be congruent (Angle-Angle-Side)

Angle bisector Given an angle, a ray that divides the angle into two equal parts

Angle Bisector Theorem The bisector of any angle of a triangle cuts the opposite side into segments whose lengths are proportional to the sides that form the angle.

ASA Criterion One of the *triangle congruence criteria*, which states that when two corresponding angles and the side that is included in between those angles have the same sizes as the corresponding angles and side of another triangle the two triangles must be congruent (Angle-Side-Angle)

Areas of similar figures If two figures are similar, then the ratio of their areas equals the square of the ratio of similarity.

Bisect Divide into two pieces that are equal

Centroid The medians of a triangle are concurrent at this point which is the balance point (also known as the center of gravity) of the triangle.

Centroid Theorem The centroid of a triangle lies on each median twice as far from the vertex as it is from the midpoint of the opposite side.

Circle This curve consists of all points that are at a constant distance from a *center.* The common distance is the *radius* of the circle.

Circumcenter The perpendicular bisectors of the sides of a triangle are concurrent at this point, which is equidistant from the vertices of the triangle.

Circumcircle When possible, the circle that goes through all of the vertices of a polygon

Collinear Three (or more) points are collinear when they all lie on a single line.

Complementary Two angles that fit together to form a right angle are called complementary. Each angle is the *complement* of the other.

Completing the Square Applied to an equation, this is an algebraic process that is useful for finding the center and radius of a circle.

Components The values that define a *vector* that describe how to move from one unspecified point to another. They are obtained by *subtracting* the coordinates.

Concurrent Three (or more) lines that go through a common point (intersect) are called *concurrent* at that point (*see also point of concurrency*)

Congruent When the points of one figure can be matched with the points of another figure so that corresponding parts have the same size, then the figures are called congruent which means that they are considered to be equivalent.

Converse The *converse* of a statement of the form “if [something] then [something else]“ is the statement “if [something else] then [something].”

Convex A polygon is called convex if every segment joining a pair of points within it lies entirely within the polygon.

Coordinates Numbers that describe the position of a point in relation to the origin of a coordinate system.

Corresponding Describes parts of figures (such as angles or segments) that have been matched by means of transformation.

Cosine ratio Given a right triangle, the cosine of one of the acute angles is the ratio of the length of the side adjacent to the angle to the length of the hypotenuse. The word cosine is a combination of complement and sine, so named because an angle is the same as the sine of the complementary angle.

Counterexample A method of proof that only proves a statement false by showing at least one example for which the statement is false.

CPCTC Corresponding Parts of Congruent Triangles are themselves Congruent.

Decagon Polygon with 10 sides

Diagonal A segment that connects two nonadjacent vertices of a polygon

Diameter A chord that goes through the center of its circle

Dilation A similarity transformation, with the special property that all lines obtained by joining points to their images are concurrent at the same central point.

Direction vector A vector that describes a line, by pointing from a point on the line to some other point on the line.

Displacement vector The displacement vector from the point (a, b) to point (c, d) is the vector $[c - a, d - b]$.

Distance formula Formula which gives the Pythagorean distance between two points in the coordinates plane. It is generally stated as the distance between (a, b) and (c, d) is $h = \sqrt{((a-c)^2 + (b-d)^2)}$.

Dodecagon Polygon with 12 sides.

Dot product Given vectors $[a, b]$ and $[m, n]$ the dot product is the number $am+bn$. When the value of the dot product is zero, the two vectors are perpendicular.

Equiangular A polygon all of whose angles are the same size.

Equidistant A shortened form of equally distant.

Equilateral A polygon all of whose sides have the same length.

Euclidean Geometry (also known as plane geometry) is characterized by its parallel postulate, which states that given a line, exactly one line can be drawn parallel to it through a point not on the given line. The Greek mathematician Euclid, who flourished about

2300 years ago, wrote many books, and established a firm logical foundation for geometry.

Euler Line The centroid, the circumcenter and the orthocenter of any triangle are all collinear. The Swiss scientist, Leonhard Euler wrote copiously on both mathematics and physics.

Exterior angle An angle that is formed by a side of a polygon and the extension of an adjacent side. It is supplementary to the adjacent interior angle.

Exterior-Angle Theorem An exterior angle of a triangle is the sum of the two non-adjacent interior angles.

Foot The point where an altitude meets the base to which it is drawn.

Function A function is a rule that describes how an input uniquely determines an output.

Glide-Reflection A transformation in a plane that leaves no single point fixed, but that does map a single line to itself. A glide-reflection maps points on either side of a line to the other side of the line. A glide-reflection is defined by a vector and a mirror line.

Greek Letters Letters from the Greek alphabet often appear in mathematics as variables representing angles and other real-life measurements. Some that are commonly used are α (alpha), β (beta), π(πι), (upper and lowercase delta) ανδ *θ (τηετα)*

Head (of a vector) Vector terminology for the second vertex of a directed segment.

Hexagon A polygon that has six sides.

H-L criterion When the hypotenuse of two right triangles have the same length, and a leg of one triangle has the same length as the leg of the other, then the triangles are congruent. The rule of evidence is known as Hypotenuse-Leg.

Image The result of applying a transformation to a point *P* is called the *image point* of *P* and is often denoted *P'*. One occasionally refers to an *image segment* of an *image triangle*.

Included Angle The angle formed by two designated segments.

Isosceles triangle A triangle that has at least two sides of the same length.

Isosceles Triangle Theorem If a triangle has two sides of equal length, then the angles opposite those sides are the same size.

Isosceles Trapezoid A trapezoid whose nonparallel sides have the same length.

Kite A quadrilateral that has two disjoint pairs of congruent adjacent sides.

Labeling conventions	Given a polygon that has more than three vertices, place the letters around the figure in the order that they are listed.
Lattice point	A point whose x-y coordinates are both integers.
Lattice rectangle	A rectangle whose x-y coordinates of all vertices are both integers.
Leg	The perpendicular sides of a right triangle are called its legs.
Length of a vector	This is the length of any segment that represents the vector. For a vector $\vec{u}$, the length of the vector is denoted by $\lvert\vec{u}\rvert$
Line of Reflection	The line that defines the translation that is a *reflection* (see *reflection*). This is the perpendicular bisector of the segment that joins a point *A*, with its reflected image *A'*.
Linear equation	Any straight line can be described an equation in the standard form of $ax + by = c$.
Magnitude of dilation	The nonnegative number obtained by dividing the length of any segment into the length of its dilation image. It is also called the *ratio of similarity* of the *dilation factor. (see also scale factor)*
Median of a triangle	Segment that joins a vertex of a triangle to the midpoint of the opposite side.
Midline of a trapezoid	This segment joins the midpoints of the non-parallel sides. Its length is the average of the lengths of the parallel sides, to which it is also parallel. Also known as the *median* in some books.
Midsegment of a triangle	The segment that connects two midpoints of the sides of a triangle. There are three midsegments per triangle.
Midsegment Theorem	A segment joining the midpoints of two sides of a triangle is parallel to the third side and is half as long.
Midpoint	The point on a segment that is equidistant from the endpoints of the segment. If the endpoints are (a, b) and (c, d), the midpoint is given by the average of the endpoints $(\frac{a+c}{2}, \frac{b+d}{2})$
Mirror	The line that defines the translation that is a *reflection* (see *reflection*).
Negative reciprocal (also called opposite reciprocal)	One number is the opposite reciprocal of another if the product of the two numbers is -1.
Octagon	A polygon that has eight sides.

Opposite	Two numbers or vectors are opposite if they differ in sign. For example, 17.5 is the opposite of -17.5, and [2,-11] is the opposite of [-2,11].
Opposite angles	In a quadrilateral, this means non-adjacent angles.
Opposite sides	In a quadrilateral, this means non-adjacent sides.
Orthocenter	The altitudes of a triangle are concurrent at this point.
Parabola	A curve consisting of those points that are equidistant from a given line and a given point is called a parabola. The equation that results in a graph that is a *parabola* is a *quadratic equation.*
Parallel	Coplanar lines that do not intersect. When drawn in a coordinate plane, they are found to have the same slope or else no slope at all.
Parallelogram	A quadrilateral that has two pairs of parallel sides.
Parameter	An additional variable that affects equations and gives further information such as time.
Pentagon	A polygon that has five sides.
Perpendicular	Coplanar lines that intersect to form a right angle. If two lines are perpendicular then their slopes are *opposite reciprocals* and therefore the product of their slopes should equal -1.
Perpendicular bisector	Given a line segment, the perpendicular bisector is the line that intersects the segment at the midpoint and is also perpendicular to that segment.
Perpendicular Bisector Property	All points that lie on the perpendicular bisector of a segment are equidistant from the endpoints of that segment.
Perpendicular vectors	Two vectors can be shown to be perpendicular if their dot product is zero.
Point of Concurrency	a point of intersection of three or more lines
Point-Slope form	A non-vertical line can be described by $y - y_1 = m(x - x_1)$ where m = the slope and (x_1, y_1) is a point that is on the line.
Postulate	A statement accepted as true without proof.
Proof by Contradiction	A method of proof that is usually called *indirect* as the desired statement to be proved in assumed false and a contradiction of a known true statement is created in the argument.
Proportion	An equation that expresses the equality of two ratios.

Pythagorean Theorem	The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs. If c represents the hypotenuse and a and b represent the legs of the right triangle, the Pythagorean equation is represented by $a^2 + b^2 = c^2$.
Quadrant	One of the four different regions that the *x* and *y* axes split the coordinate plane into. Quadrant 1 is where both the *x* and *y* coordinates are positive and then they are numbered consecutively counterclockwise generally using roman numerals.
Quadratic Formula	The formula that is used to solve for the x-intercepts of a quadratic equation in standard form $y = ax^2 + bx + c$. The quadratic formula is stated as $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Quadrilateral	Polygon with four sides.
Ratio of similarity	The ratio of any two corresponding sides of a pair of similar polygons.
Reflection	A transformation in a plane that has a line of fixed points, which is the mirror of reflection. All other points are mapped to a point that is equidistant to the mirror line and perpendicular to it.
Regular	A polygon that is equilateral and equiangular.
Rhombus	An equilateral quadrilateral.
Right Angle	An angle that measures 90 degrees.
Right Triangle Similarity Theorem	In a right triangle, the altitude drawn to the hypotenuse created three similar right triangles.
Rotation	A transformation in the plane that leaves a single point fixed (called the center of rotation).
Scalar	In the context of vectors, this is the value by which the vector is multiplied/divided in order to scale it up/down (i.e. to change its length).
Scale Factor	This is the ratio in which the sides of two similar polygons are related. In a dilation, it is also the size by which the size enlarge or decrease in size (*see also dilation factor or magnitude of dilation)*
Scalene	A triangle with all three sides of different length.
Segment	The part of a line between two points.
Sentry Theorem	The sum of the exterior angles of a polygon is equal to 360 degrees.

Shared-Altitude Theorem	If two triangles share the same altitude, the ratio of their areas is equal to the ratio of their bases.
Shared-Bases Theorem	If two triangles share the same base, the ratio of their areas is equal to the ratio of their altitudes.
SAS Criterion	If two sides and the included angle between them in one triangle are congruent to the corresponding sides and included angle of another triangle, then the two triangles are congruent. This is the *Side-Angle-Side* Criterion.
SSA	This combination is not sufficient evidence to prove that two triangles are congruent. Side-Side-Angle does not guarantee a unique triangle.
SSS Criterion	If all three sides of one triangle are congruent to all three sides of another triangle, then the triangles must be congruent. This is the *Side-Side-Side* Criterion.
Similar	Two polygons are similar if their sides can be paired up in such a way that all sides are in a constant proportion, known as the *ratio of similarity*. All corresponding angles of similar figures are congruent.
Sine ratio	Given a right triangle, the sine of one of the acute angles is ratio of the opposite side to that angle to the hypotenuse. Its value can also be interpreted as what percentage of the hypotenuse the opposite side is (that's why it's always less than 1)
Skew lines	Two lines are skew if they are noncoplanar – this effectively means they do not intersect and are not in the same plane.
Slope	The rate of change of a linear graph. It can be found by dividing the change in corresponding y coordinates by the change in corresponding x coordinates.
Slope-intercept form	The form of the equation of line that gives the information of the slope and the y-intercept, $y = mx + b$.
Supplementary	Two angles that fit together to form a straight line.
Tail	In vector terminology, the tail is the point at which the vector begins.
Tail-to-tail	In vector terminology, for two directed segments with the same initial point.
Tangent ratio	Given a right triangle, the tangent of one of the acute angles is equal to the ratio of the opposite side from the angle to the adjacent side to the angle.
Tangent and slope	When an angle is formed by the positive x-axis and a ray through the origin, the tangent of the angle is the slope of the ray. Angles are measured in counterclockwise direction from the x-axis.

Tessellate	To fit non-overlapping figures together to entirely cover a plane.
Tetrahedron	A regular triangular pyramid.
Three Parallels Theorem	Given Three (or more) parallel lines, the segments that are cut on one transversal will be proportional to the segments that are cut on any transversal.
Translation	A transformation in the plane that takes a figure in the plane and uses a vector to move the figure a certain distance and direction.
Transformation	A geometric or algebraic change of an object that fits a pattern for all parts of the object. Some transformations that are common are reflections, rotations, translations and dilations.
Transversal	A line that intersects with two other lines in a diagram.
Trapezoid	A quadrilateral with exactly one pair of parallel sides. If the two non-parallel sides are congruent the trapezoid is called *isosceles.*
Triangle Inequality Theorem	Any side of a triangle is less than or equal to the sum of the other two sides.
Unit Circle	The circle that is centered at the origin with a radius of 1. It is used in finding the sine, cosine and tangent of angles and studying advanced topics of trigonometry.
Varignon Quadrilateral	Given any quadrilateral, the Varignon quarilateral is formed by connecting the midpoints of the sides of the quadrilateral.
Vector	A directed segment. A vector defined by its direction, length (magnitude), and slope while a segment has only its length.
Velocity	Velocity is a rate of change or speed of an object that has direction.
Vertex	The vertex of a polygon in a plane (or polyhedra in the three-dimensions) the one-dimensional point where two sides (edges) come together, or three faces come together.
Vertical angles	Angles that are formed by two intersecting lines that are non-adjacent.